# STATISTICAL CONTROL OF A PROCESS SUBJECT TO TOOL WEAR

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**Abstract**

Many production processes are subject to tool wear. In this case, the process means changes systematically and an upward or a downward trend is observed in the control chart. Such processes must be regularly adjusted. When samples are taken from highly capable processes, the within-sample variability is considerably less than the allowable variability given by the specification limits. If the classical Shewhart control chart is used, some points may lie outside control limits giving a signal that an assignable cause exists. Elimination of this special cause is not feasible from an economic point of view and the cutting tool is not replaced as long as the output remains within the specifications. In this case modified control limits or acceptance control charts can be used. A further refinement is to model the trend and control for assignable causes that might change the trend. The approach is applied to the manufacturing process in the automotive sector, namely to machining stoppers used in automotive transmission systems. The modified and regression charts are designed based on the retrospective analysis and a change of the actual process setup allowing prolonging the tool's operational life is recommended.

**Keywords**: modified control chart, regression control chart, process capability, machining process

## 1. INTRODUCTION

Statistical process control (SPC) is a widely adopted technique for ensuring product quality. It belongs to main management tools within ISO/TS 16949 standard requirements and the use of SPC is stated in customer-specific requirements manuals. Based on regular monitoring and timely detecting special causes of increased variation, the process can be adjusted before problems may pass to the next phase of production. As a result, operational costs are reduced.

SPC is based on distinguishing two sources of variation. Small unavoidable causes result in natural variability that is always present, but occasionally some special causes, e.g. an improperly adjusted machine tool, inferior raw material or operator errors may arise. They shift the process level and thus increase the overall output variation. A major objective of SPC is to detect their occurrence as soon as possible so that necessary corrections may be carried out. A process operating without special causes is said to be in statistical control. This state is a precondition for assessing the process capability to meet customer requirements that are often specified in form of tolerance limits. Usually the inherent process variation cannot be reduced, but adjusting of the process level can sometimes improve the process capability.

Control charts are used for this purpose. Since the year 1926 when W. A. Shewhart developed the first control chart, many other control charts have appeared, but classical Shewhart charts have been the most used charts in practice ever since. The main reason is definitely their simplicity. They can perform well in processes with not so very high capability and with constant level, but they usually fail in case of a process with high capability or in processes where some shifts of the mean are not removable either from technical or economic reasons. A typical example is a process with trend arising due to tool wear.

Modified control chart, see e.g. [1], [2], [3] or an acceptance control chart, see e.g. [4] or [5], can be used instead of the Shewhart control chart. An adaptive acceptance control chart developed especially for processes subject to tool wear was proposed in [6]. The modified and acceptance charts differ from the Shewhart chart by a distance of control limits. Unavoidable drift of the mean is considered a part of inherent variation and the control limits are more distant from the center line to allow for this drift. A process with trend, where the mean changes systematically, represents a special case and beside modified limits, a model of the trend can be utilized. Manuele [7] introduced a tool-wear chart using modified limits and regression techniques. The regression charts are also used in [8] and [9].

In this paper a problem of a real machining process control is analyzed. The modified and regression charts are designed based on retrospective analysis and the process adjustment allowing extending the tool's operational life is recommended. Besides, applicability of common capability or performance indices to this process is discussed.

## 2. CONTROL CHARTS FOR A PROCESS WITH TREND

### 2.1 Modified control chart

The aim of a modified control chart is to determine whether the process mean is within acceptable limits such that some specified (acceptable) proportion $\delta$ of nonconforming items is not exceeded. The acceptable limits *APL* are based on technical or economic grounds or on minimum required process capability. If the specification limits *USL* and *LSL* are known, as well as the underlying distribution of the process output, the *APL*s may be defined in terms of an acceptable proportion (or percentage) $\delta$ of nonconforming items which would occur if the process was centered at the *APL*. If the underlying distribution is normal, the upper and lower acceptable limits for the mean are

$$APL_U = USL - z_\delta \hat{\sigma}, \quad APL_L = LSL + z_\delta \hat{\sigma} \tag{1}$$

where $z_\delta$ is an upper normal percentile and $\hat{\sigma}$ denotes an estimate of the process standard deviation. Often $\hat{\sigma} = \bar{R} / d_2$ is used, where $\bar{R}$ denotes the average range of samples from the process and $d_2$ is tabulated for various sample sizes, e.g. in [10]. Other methods of estimation known from SPC can be chosen.

Several possibilities of determining modified control limits exist. Commonly, limits are given by

$$UCL = USL - z_\delta \hat{\sigma} + \frac{z_\alpha \hat{\sigma}}{\sqrt{n}}, \quad LCL = LSL + z_\delta \hat{\sigma} - \frac{z_\alpha \hat{\sigma}}{\sqrt{n}} \tag{2}$$

where $\alpha$ denotes a type I error probability (risk of false alarm), see e.g. [1]. Bissell [11] recommends to choose $\delta = \alpha = 0{,}00135$, i.e. $z_\delta = z_\alpha = 3$.

According to Hill [12], see also [3], the modified limits should be placed inside the acceptable limits for the mean, i.e.

$$UCL = USL - z_\delta \hat{\sigma} - \frac{z_\beta \hat{\sigma}}{\sqrt{n}}, \quad LCL = LSL + z_\delta \hat{\sigma} + \frac{z_\beta \hat{\sigma}}{\sqrt{n}} \tag{3}$$

where $\beta$ denotes a type II error probability (risk of not taking action). Hill recommends $\beta = 0.05$, i.e. $z_\beta = 1.645$. If these limits are narrower than classical Shewhart limits, the classical limits should be used.

Dietrich and Schulze [13] suggest three other methods for calculation of modified limits. The first method takes into account two sources of variation, within-group variation measured by $\sigma$ and variation of subgroup means measured by $\sigma_A$:

$$UCL = \hat{\mu} + \frac{3\hat{\sigma}}{\sqrt{n}} + 1.5\,\hat{\sigma}_A, \quad LCL = \hat{\mu} - \frac{3\hat{\sigma}}{\sqrt{n}} - 1.5\,\hat{\sigma}_A, \tag{4}$$

where $\hat{\mu} = \bar{\bar{x}}$ is the overall average and $\hat{\sigma}^2$ and $\hat{\sigma}_A^2$ are obtained using mean squares in ANOVA table when the random effects one-way ANOVA model is considered. Details can be found in [13] or in any book on experimental design, see e.g. [14].

The second approach uses standard deviation of averages. Control limits are

$$UCL = \hat{\mu} + 3\,\hat{\sigma}_{\bar{x}}\,, \quad LCL = \hat{\mu} - 3\,\hat{\sigma}_{\bar{x}}\,, \quad \text{where } \hat{\sigma}_{\bar{x}} = s_{\bar{x}} = \sqrt{\frac{1}{k-1}\sum_{i=1}^{k}(\bar{x}_i - \bar{\bar{x}})^2}\,. \tag{5}$$

The third type of limits is based on the overall standard deviation $\sigma_{tot}$ comprising both within-group variation and variation of means:

$$UCL = \hat{\mu} + \frac{3}{\sqrt{n}}\hat{\sigma}_{tot}\,, \quad LCL = \hat{\mu} - \frac{3}{\sqrt{n}}\hat{\sigma}_{tot}\,, \quad \text{where} \quad \hat{\sigma}_{tot} = \sqrt{\frac{1}{kn-1}\sum_{i=1}^{k}\sum_{j=1}^{n}(x_{ij} - \bar{\bar{x}})^2}\,. \tag{6}$$

The assumption of normality is substantial in all the methods, especially in (1) to (3), because the location of control limits depends rather heavily on it. Observations as a whole represent a mixture of distributions with means changing in a systematic manner over time. Control limits are based on "short-time" distributions with threshold means given by (1) in case of a normal distribution. If the short-time distribution was not normal, percentiles $z_\delta$, $z_\alpha$, and $z_\beta$ would have to be replaced accordingly. A special attention must be given to checking for normality, because even if short-time distributions are normal, their mixture is not and so residuals obtained from a model of trend should be used for testing.

## 2.2 Regression control chart

It is assumed that if the process subject to tool wear is "in control", i.e. no other causes exist, the wear rate is constant. The change of this rate, especially its increase, is undesirable. To check for special causes other than the unavoidable tool wear, the regression chart by Mandel [8] can be used. The chart is based on the model of trend. Usually a regression line is used as the first approximation, although the method can be modified to account for other trend models. When subgroups are taken from the process at regular intervals, the regression line can be expressed by means of

$$Y = b_0 + b_1 i\,, \tag{7}$$

where $i$ is the sample number and $b_0$ and $b_1$ are obtained by the least squares method.

Two forms of displaying are possible. The center line represented by the regression line can be added to the modified or acceptance control chart; then two parallel sloping control limits are given by

$$UCL = b_0 + b_1 i + L\frac{\hat{\sigma}}{\sqrt{n}}\,, \quad LCL = b_0 + b_1 i - L\frac{\hat{\sigma}}{\sqrt{n}} \tag{8}$$

These control limits differ from confidence limits used in regression analysis, see e.g. [15]. Mandel [8] gave reasons for using parallel limits (8) with $L$ = 2, Mitra [1] presents equations (8) with $L$ = 3.

The other possibility is to remove the trend from data and construct a separate $\bar{X}$ - chart for residuals. Then the center line goes through zero and control limits are at $\pm L\hat{\sigma}/\sqrt{n}$ from it. The process standard deviation $\sigma$ can be estimated by means of the regression model, too.

## 2.3 Process performance

The use of common capability or performance indices for processes subject to tool wear is contentious. A process with trend is not in statistical control, strictly speaking. Construction of these indices is based upon an idea that the target lies in the center of specifications and the mean is constant. They give no idea about the fraction of nonconforming items in case of a systematic mean's shift and they seem useless in connection with the modified control chart where the fraction nonconforming is guaranteed. Nevertheless, they give some picture about the process and so they are considered in the paper.

Capability indices

$$C_p = \frac{USL - LSL}{6\sigma} \quad \text{or} \quad C_{pk} = \frac{\min\{USL - \mu, \mu - LSL\}}{3\sigma} \tag{9}$$

and performance indices

$$P_p = \frac{USL - LSL}{6\sigma_{tot}} \;, \quad P_{pk} = \frac{\min\{USL - \mu, \mu - LSL\}}{3\sigma_{tot}} \tag{10}$$

are commonly used in practice.

Although in [16] or [17] some modification is proposed at which a part of the overall variability is estimated based on the trend model, the index has not become established.

Another index proposed by Hsiang and Taguchi in 1985, see also [2] or [18] may be of some interest in cases where the target is not located in the center of specifications:

$$C_{pm} = \frac{C_p}{\sqrt{1 + \left(\frac{\mu - T}{\sigma}\right)^2}} \;, \quad C_{pkm} = \frac{C_{pk}}{\sqrt{1 + \left(\frac{\mu - T}{\sigma}\right)^2}} \tag{11}$$

The target value can be set equal to the nominal value or a mid-tolerance value, e.g. $APL_L$ given by (1).

## 3. PROCESS OF MACHINING

Methods described above are applied to statistical control of a manufacturing process from the automotive sector.

### 3.1 Process description

A company manufactures metal components that are supplied to a producer of automotive transmissions. These components (stoppers) are made from cast iron by machining process. The stopper is held in a fixture and cut by a tool. Machined components contain particles of different kinds; their hardness may be comparable with hardness of the tool and an abrasive effect appears. Furthermore, a large amount of heat develops on the surface of the tool during the cutting process. These abrasive and thermal factors cause the tool wear.

The width of machined components is an important quality characteristic; the nominal value is 7 mm and specification limits are +0.15/-0 mm, that means the lower limit of 7 mm and the upper limit of 7.15 mm. It is measured by a caliper with resolution of 0.01 mm. Due to cutting wear during the machining process the width of stoppers gradually increases. Before the width reaches the upper specification, the cutting tool must be replaced by a new one. **Fig. 1** displays a typical pattern of such regularly adjusted processes.

Two requirements must be taken into account to adjust the process properly. Firstly, specifications must be satisfied, secondly, tool's operational life should be as long as possible. The latter requirement implies that the machine program must be set up as closely to the lower specification as possible, but on the other hand sufficiently far from it, so that the stopper width cannot exceed the lower specification. For the meantime, the company uses the classical Shewhart control $\overline{X}$-chart and *R*-chart in SPC. Subgroups of size 5 are taken from the process and their sample characteristics, i.e. averages and ranges, are drawn in the charts.

The chart with limits constructed based on data from one cycle between two process adjustments is displayed in **Fig. 2**. It appears that due to high process capability the Shewhart control limits are quite close to the central line. To obey the chart, tool's operational life is unnecessarily short.

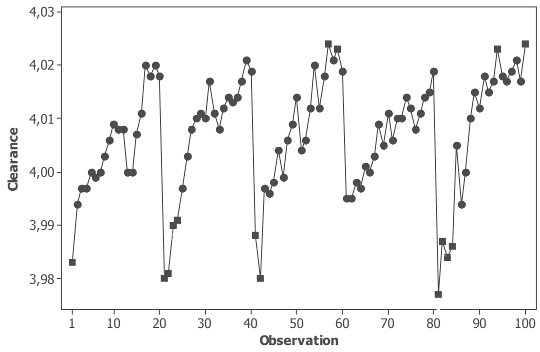


**Fig. 1** Regularly adjusted process, illustrative data

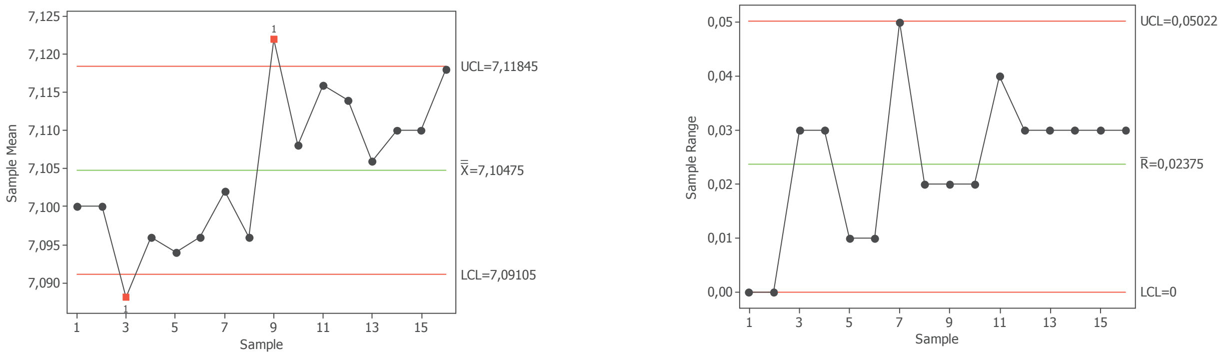


**Fig. 2** Shewhart control charts for averages and ranges

### 3.2 Alternative control chart design

A modified chart with limits according to (2) was chosen to solve the problem. The estimated process standard deviation $\hat{\sigma}$ is 0.01021 and the risk of false alarm $\alpha$ = 0.0027 was chosen as usual. The acceptable fraction nonconforming $\delta$ was derived from the target capability index $C_p$ = 1.67, entailing 233 ppm defective at maximum (see e.g. [2]), from where $\delta$ = 0.000233. The acceptable limits for the mean and the modified control limits according to (2) are

$$APL_U = 7.114 \qquad UCL = 7.128$$

$$APL_L = 7.036 \qquad LCL = 7.022$$

The fitted regression line $Y$ = 7.09205 + 0.00149 $i$ is displayed in **Fig. 3** together with two pairs of sloping control limits. The internal limits correspond to the equations

$$Y + 2\hat{\sigma} = 7.112 + 0.00149\, i \qquad Y - 2\hat{\sigma} = 7.072 + 0.00149\, i$$

For illustration, regression prediction intervals represented by the external limits are added in **Fig. 3**.

Although no subgroup average crossed the horizontal control limits and the process can be considered "in-control" in view of the modified control chart, it appears that the actual machining process setup is quite distant from the target. As for the sloping limits, the 9th subgroup average lies outside the upper one but since the process returned back inside the limits without any intervention, apparently a false signal occurred.

Based on the analysis, the way of the machining process' adjustment should be changed. The target value at the setup should be $APL_L$ = 7.036 mm. In this way the operational life will be considerably (more than twice according to trial data) prolonged. Horizontal control limits at 7.022 mm and 7.128 mm for averages can be used for ongoing control. After each adjustment the sloping control limits can be drawn and the retrospective analysis performed. For the short-time variation control the classical R-chart can be used.

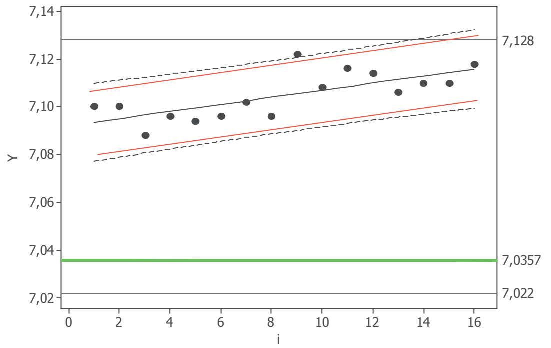


**Fig. 3** Modified control chart for averages with sloping control limits

### 3.3 Process performance

Various capability and performance indices are displayed in **Table 1**. The process capability $C_p$ = 2.45 is high enough to relax control limits and apply some type of modified control charts. The process performance measured by $P_p$ = 1.746 is sufficient but $P_{pk} = P_{pkU}$ = 1.053 indicates that the process mean is nearer to the upper specification. It is not convenient with regard to the nominal value and apparently, the process is not properly adjusted. The large distance between the mean and the nominal value is reflected by small values of $C_{pm}$ or $P_{pm}$. Obviously, the process adjustment recommended above will make $P_p$ lower but as was pointed out earlier, the value of $P_p$ is not decisive in case when the process mean systematically increases. Construction of modified limits guarantees the fraction nonconforming corresponding to $C_p$ = 1.67 when the limits are not exceeded.

**Table 1** Estimated capability and performance indices

| Index | Estimate | Index | Estimate | Index | Estimate | Index | Estimate |
|---|---|---|---|---|---|---|---|
| $C_p$ | 2.448 | $C_{pk,U}$ | 1.477 | $C_{pk\,L}$ | 3.420 | $C_{pk}$ | 1.477 |
| $P_p$ | 1.746 | $P_{pk,U}$ | 1.053 | $P_{pk\,L}$ | 2.439 | $P_{pk}$ | 1.053 |
| $C_{pm}$ | 0.2375 | $C_{pkm\,U}$ | 0.143 | $C_{pkm\,L}$ | 0.332 | $C_{pkm}$ | 0.143 |
| $P_{pm}$ | 0.2365 | $P_{pkm\,U}$ | 0.143 | $P_{pkm\,L}$ | 0.330 | $P_{pkm}$ | 0.143 |

### 3.4 Checking for normality

Results of normality tests are shown in **Table 2**. The tests were applied both on original data and on residuals obtained from the regression model of the trend. Expectedly, the residuals tend to be more "normal" according to most of the tests. Low p-values (less than 0.05) indicating normality violation are attributed to insufficient capability of the measurement system in the first place. It leads to repeated rounded values, as can be seen in **Fig. 4**. The Ryan-Joiner test is robust in this respect and that is why it does not reject normality. Obviously, caliper's resolution of 0.01 is rather poor. Nevertheless, this resolution is more than 10x less than the distance between specifications and so the gage is considered acceptable.

**Table 2** Normality tests

| Normality test | Original values | | Residuals | |
|---|---|---|---|---|
| | Statistic | P-value | Statistic | P-value |
| Anderson-Darling | 3.949 | < 0.005 | 1.029 | 0.01 |
| Ryan-Joiner | 0.990 | > 0.1 | 0.986 | 0.075 |
| Shapiro-Wilk | 0.889 | < 0.001 | 0.953 | 0.014 |

Normal probability plots representing the graphical method of normality checking are displayed in **Fig. 4** and **5**. Associated confidence intervals enable to assess the nature of normality violation. As was remarked above, **Fig. 4** implies that the rounded values may be the main reason. The fact that most of the points in **Fig. 5** lie within confidence limits indicates that the violation of normality is not so serious.

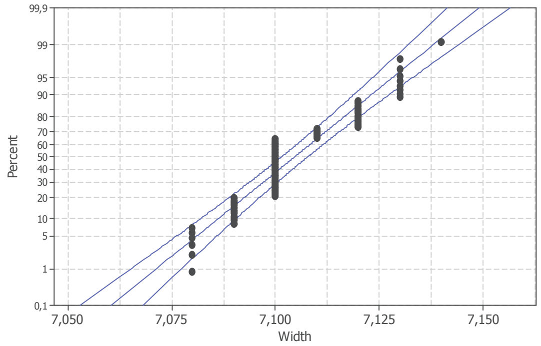


**Fig. 4** Normal probability plot, original data

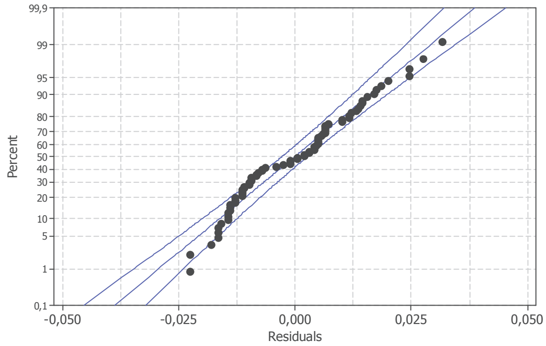


**Fig. 5** Normal probability plot, residuals

## CONCLUSION

Modified or acceptance control charts allow for a non removable trend in tool-wear processes and an apparent economical effect arises due to prolongation of the tool's operational life. The analysis of the machining process revealed the actual non-economical process-adjustment policy and outlined an opportunity for reducing costs associated with frequent tool replacements. Sloping control limits supplemented after each cycle of process adjustment represent another refinement and can be used for a retrospective analysis. If the pattern of wear repeats regularly, the sloping center line with parallel control limits can be drawn in advance and on-line control of the process with respect to its trend is possible. Intersections of the centerline with horizontal lines representing acceptable limits for the process mean can be used to predict the tool's operational life.

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