AN EXACT DECISION-MAKING IN PRODUCTION BASED ON MULTICRITERIA OPTIMIZATION

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Abstract

As the competition gets stiffer, it is imperative for many companies to stay competitive. To do so, companies must fine-tune their products in terms of what these products offer to the customer, i.e. what product features are supplied to the market. If the features can be quantified, as is typical in industry, particularly in metallurgy, mathematical methods are at hand to suggest in an exact way how to improve the product features so that they are closer to an ideal state. Some of these methods can be based on the mathematical concept of distance, which is one of the most important concepts in the world of science and technology. This concept is presented in the paper, and is used to suggest how companies can improve their competitiveness in a cost-effective way.

Keywords: competitiveness, multicriteria decision-making

1. INTRODUCTION

If a company registers that sales of a competitive product exceed its own sales, it should, apart from improving its management system [5], run a search on what features the competitive product offers to the customer, and compare the level of its own product features to the level of the competitive product features to see relevant differences. Such a search is usually easy to do, and if the features are technically sophisticated, various scholarly journals often provide customers with results of consumer product tests that are detailed enough to get a clear view of the product standards. Let us have a comparison of two products, based on a consumer test, for instance. Let one of the products is manufactured by a company under scrutiny that we shall call „company A“ for further references in this paper, while the other product belongs to a competitive company, henceforth referred to as „company C“. If the character of the product we are talking about was such that the product was represented by a single feature, it would be simple for the company A to determine what to do with its own product in the future to keep up its existing position in the market: it should simply improve the level of that single feature for its own product so that it was at least as good as the competitor, the company C. The problem arises when products are depicted by more that one feature, since it happens then that two products of the same type are incomparable in terms of the levels of the features they provide to the customer - one product is often better than another in one respect, but simultaneously it is also worse than the other product in another respect. If this is the case, how should the company that falls behind its competitor change its product features to catch up with the competitor? Generally speaking, there could be an infinite number of ways how to change the product features, and draw closer to the competition.

To give an example of the problem just outlined, let us have a two-feature product - a computer machine. The customer is interested in the performance of the machine, represented by its chip frequency measured in gigahertz, and the color of the machine measured, say, in wavelengths. Staying in an interval of values, we can theoretically change the color and the chip frequency in an infinite number of ways to attract the customer - we can raise the brightness of the color to any extent we want, and the same is true about the frequency. What is the proper combination of these changes to become competitive? Of course, no one will ever know exactly how the customer behaves. Not even the customers themselves often know why they buy this product and not the other product, because their behavior is affected by a *proper mix* of the product features that they cannot describe exactly. But do we need to have that insight? Not really, surprisingly. All we need to know is which product is becoming competitive in the market. Such a product, belonging to the company C, for instance, becomes a standard for our product, for the product of the company A, for example. And we are not so interested in the explanation why the company C's product is the one the company A's product wants to resemble at that moment.

All that is necessary is to resemble the good product, within legal limits, of course, the explanation why playing no role. There are two questions, however, that must be resolved in this respect. The first question relates to what the word „resemble“ means. The second question is how to resemble another company's product without spending much money on it, i.e. how to make the resemblance as cheap as possible.

These two questions are what this paper aims to resolve in an *exact* manner. The question number one has to do with distance [1], and this is the concept incorporated in this paper to solve the outlined problems. The second question has to do with optimization. We present a procedure how to perform the optimization *exactly* and *explicitly*, the latter term meaning that our result is an explicit formula to be used to get the optimal changes in the product features. It is possible to perform the optimization, using a proper software, without the necessity to have knowledge of advanced mathematics. It must be stressed, however, that computer optimization software uses a *fixed* algorithm which may not give expected results. The problem with any optimization software is that the algorithm used is such that it assumes certain mathematical conditions to be valid, which may not be the case. If the conditions are not valid, the software may return a wrong result. And this is where the results of this paper should be used, because we prove that the solution presented here is the true optimal solution.

2. DISTANCE AS A RESEMBLANCE

Let *n* product features $(x_1, x_2, \dots, x_n)$ be observed by the customer. For the competitive product of the company C, let the features attain specific levels $(y_1, y_2, \dots, y_n)$, whereas for the product of the company A, let these levels be $(z_1, z_2, \dots, z_n)$. These levels can be grades generated by a consumer product test, for instance, the grades usually having the property that the higher the mark, the better the corresponding feature. This property will be assumed to hold in our analysis. There is usually also an idea what levels are the most desirable within technical and capabilities of a company. Let these desirable levels be $(\max_1, \max_2, \dots, \max_n)$. Further, our situation is assumed to be such that the company C experiences higher sales of its product than the company A, and thus the company A wishes to change its product features to be as competitive as the company C. A wise strategy is to change the features so that the changed product resembles the competitive product, drawing customers' attention. It should be similar in what it offers to the customer, although for legal purposes, it cannot be identical. One way how to proceed is to measure how far the company C's product is from the desirable situation, and set up the company A's product features in such a way that its product will be as far away from the desirable state, represented by the vector $(\max_1, \max_2, \dots, \max_n)$, as the company C's product. However, there are many ways how to make this set-up, and we specifically search for the one that will cost the company A the least. The two products having the same distance from the desired state, such a move can be regarded as making the two products as similar as possible. Thus, the distance can be viewed in this context as a measure of resemblance of the two products. We avoid obvious results by assuming that $(y_1, y_2, \dots, y_n) \neq (\max_1, \max_2, \dots, \max_n)$.

To sum up, our goal is to find features of the company A's product that would render the product as desirable as the company C's product, the desire being measured by how far a product is from the most optimal levels given by $(\max_1, \max_2, \dots, \max_n)$. And we want to achieve this objective cheaply. Although there is an infinite number of ways how to measure distance in mathematics, we shall use the distance used perhaps most frequently. It is the Euclidean distance [2]. Given two vectors $\mathbf{x}=(x_1, x_2, \dots, x_n)$ and $\mathbf{y}=(y_1, y_2, \dots, y_n)$, or given two points in an *n*-dimensional real Euclidean space, to be precise, the Euclidean distance between the two points $\rho(\mathbf{x},\mathbf{y})$ is

$$\rho(\mathbf{x},\mathbf{y})=\left(\sum_{i=1}^{n}(x_i-y_i)^2\right)^{1/2}. \tag{1}$$

Companies know how much it costs to change their product feature. For the *i*-th feature, let the cost be $P_i>0$. Thus, to change the current *i*-th product feature level x_i to a competitive or desired level a_i, we assume the company A needs to spend P_i crowns, dollars or any other currency units it uses in its business. Using this financial measure, what we seek is a solution to the following optimization problem

$$\min_{x_1, x_2, \dots, x_n} P_1(x_1-z_1)+P_2(x_2-z_2)+\dots+P_n(x_n-z_n) \tag{2}$$

subject to the condition

$$\left(\sum_{i=1}^{n}(x_i-\max_i)^2\right)^{1/2}=\left(\sum_{i=1}^{n}(y_i-\max_i)^2\right)^{1/2}. \tag{3}$$

The function in (2) measures the total costs the company A will record when changing its product features from the levels $(z_1, z_2, \dots, z_n)$ to new levels $(x_1, x_2, \dots, x_n)$. Since the function is minimized subject to (3) if and only if the function $P_1x_1+P_2x_2+\dots+P_nx_n$ is minimized subject to

$$\sum_{i=1}^{n}(x_i-\max_i)^2=\sum_{i=1}^{n}(y_i-\max_i)^2, \tag{4}$$

we shall keep the analysis simpler, dropping the z_i's in our optimization problem. Therefore, we shall work with a simpler, but equivalent problem

$$\min_{x_1, x_2, \dots, x_n} P_1x_1+P_2x_2+\dots+P_nx_n \tag{5}$$

subject to the condition

$$\sum_{i=1}^{n}(x_i-\max_i)^2=\sum_{i=1}^{n}(y_i-\max_i)^2. \tag{6}$$

The right-hand side of (6) is a given number, a constant we are going to denote *c*.

3. OPTIMIZATION

Let us now solve explicitly the problem defined by (5) and (6). To solve the problem, it is first necessary to show the solution exists. Showing that the function in (5) is continuous and the set

$$M=\left\{(x_1, x_2, \dots, x_n)\in R^n : \sum_{i=1}^{n}(x_i-\max_i)^2=c\right\} \tag{7}$$

is compact, which is equivalent to saying the set is closed and bounded in R^n, is one familiar way of proving the solution exists. Since the continuity of the linear function in (5) everywhere in R^n is a trivial and well-known fact, we shall focus on the properties of *M*.

To prove that *M* is closed, we must show that every point which has a zero distance from *M* belongs to *M*. To show this, we use two well-known facts: the fact that a point $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ has a zero distance from *M* if and only if there exists a sequence of points $\{\mathbf{x}_k\}_{k=1}^{\infty}$ from *M*, where $\mathbf{x}_k = (x_{1,k}, x_{2,k}, \ldots, x_{n,k})$, such that $\lim_{k\to\infty} \rho(\mathbf{x}_k, \mathbf{x}) = 0$; and the fact that a real function $f(\mathbf{x}) = f(x_1, x_2, \ldots, x_n)$ is continuous in R^n if and only if for any point $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in R^n$ and any sequence of points $\{\mathbf{x}_k\}_{k=1}^{\infty}$ from R^n, for which $\lim_{k\to\infty} \rho(\mathbf{x}_k, \mathbf{x}) = 0$, the limit $\lim_{k\to\infty} f(\mathbf{x}_k) = f(\mathbf{x})$ holds.

Let us have a point $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ with a zero distance from M. Let $\{\mathbf{x}_k\}_{k=1}^{\infty}$ be a sequence of points from *M* such that $\lim_{k\to\infty} \rho(\mathbf{x}_k, \mathbf{x}) = 0$. Since the points $\mathbf{x}_k$ are from *M*, $\sum_{i=1}^{n} (x_{i,k} - \max_i)^2 - c = 0$ for each integer *k*. Since $f(\mathbf{x}) = \sum_{i=1}^{n} (x_i - \max_i)^2 - c$ is a multivariate polynomial, and thus a continuous map everywhere in R^n, we have

$$\sum_{i=1}^{n} (x_i - \max_i)^2 - c = \qquad (8)$$

$$= f(\mathbf{x}) = \lim_{k\to\infty} f(\mathbf{x}_k) = \lim_{k\to\infty} \sum_{i=1}^{n} (x_{i,k} - \max_i)^2 - c = 0.$$

Thus, the point $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ belongs to *M*, which was to be proved. The set *M* is therefore closed.

Let us now turn our attention to whether *M* is bounded. To prove this, it suffices to show that for *any* two points $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in M$, $\mathbf{y} = (y_1, y_2, \ldots, y_n) \in M$, their Euclidean distance $\rho(\mathbf{x}, \mathbf{y})$, or its second power $\rho^2(\mathbf{x}, \mathbf{y})$, is bounded. It then follows that

$$\sup_{\mathbf{x}\in M,\, \mathbf{y}\in M} \rho(\mathbf{x}, \mathbf{y}) < \infty, \qquad (9)$$

which is the definition of boundedness of a set. For any two points $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in M$, $\mathbf{y} = (y_1, y_2, \ldots, y_n) \in M$, we have

$$\rho^2(\mathbf{x}, \mathbf{y}) = \sum_i (x_i - y_i)^2 = \qquad (10)$$

$$= \sum_i (x_i - \max_i)^2 + \sum_i (\max_i - y_i)^2 + 2\sum_i (x_i - \max_i)(\max_i - y_i),$$

or

$$\rho^2(\mathbf{x}, \mathbf{y}) - 2c = 2\sum_i (x_i - \max_i)(\max_i - y_i). \qquad (11)$$

Thus

$$\left|\rho^2(\mathbf{x}, \mathbf{y})\right| = \left|2c + 2\sum_i (x_i - \max_i)(\max_i - y_i)\right| \qquad (12)$$

$$\le 2c + 2\sum_i \left|(x_i - \max_i)(\max_i - y_i)\right| \le 2c + 2nc^2.$$

M is therefore bounded as well, which completes the proof of its compactness.

Knowing that our constrained optimization problem given by (5) and (6) has a solution, we may find the solution, using the technique of Lagrange multipliers. The solution $\mathbf{a}=(a_1, a_2, \dots, a_n)\in M$ satisfies the following set of equations

$$P_i + 2\lambda(a_i - \max_i) = 0, \quad i = 1, 2, \dots, n, \tag{13}$$

where is λ is the Lagrange multiplier, and also the feasibility equation

$$\sum_{i=1}^{n}(a_i - \max_i)^2 - c = 0. \tag{14}$$

From (13), we get

$$a_i = \frac{2\lambda \max_i - P_i}{2\lambda}, \quad i = 1, 2, \dots, n, \tag{15}$$

which, if inserted in (14), gives

$$\lambda = \pm\frac{\sqrt{\sum_{i=1}^{n} P_i^2}}{2\sqrt{c}} \tag{16}$$

and therefore, reinserting (16) with the plus sign to (15),

$$a_i = \max_i - \frac{P_i\sqrt{c}}{\sqrt{\sum_{i=1}^{n} P_i^2}}, \quad i = 1, 2, \dots, n. \tag{17}$$

Reinserting (16) with the minus sign to (15) would obviously increase the cost function (5), and cannot thus lead to the minimization of the function. Substituting (17) for x_i in (6) shows that the solution (17) does indeed satisfy the condition (6).

Two notes are in place, regarding the solution (17). First, as can be seen from (17), the higher the cost P_i of adjusting the *i*-th product feature to the level a_i, the smaller the level of that product feature will be. In other words, it will not be financially convenient for the company A to raise the level of this feature too much simply because doing so would be too expensive for the company. It is obvious that the result (17) may also suggest to lower the value of the *i*-th product feature so that this lowering may be too steep to be realizable at all. For instance, the result (17) may lead to the *i*-th feature being negative, which is in reality impossible. Therefore, the result is applicable in cases where the costs P_i are not extremely high. Secondly, as can be seen from (17) again, the result confirms what many companies do these days - to be competitive, they improve those product features that are cheap to improve.

4. EXAMPLES

Some theoretical approaches were presented in the previous sections, and it is time to show how to work with the results. We do so now. Two examples are presented in this section. The first example shows that our theoretical result is in line with the result returned by one of the more frequently used optimization software packages - the Excel Solver module. The second example shows why it is more convenient to have an explicit result at hand, since the Solver may return an *approximate* solution which is not exactly optimal because it is not feasible in the first place - it does not satisfy requirement (6).

4.1. Example 1

Let us have for illustrative purposes the setting: $\mathbf{max} = (\max_1, \max_2) = (10, 10)$, $\mathbf{P} = (P_1, P_2) = (5, 4)$, $\mathbf{z} = (z_1, z_2) = (6, 7)$, $\mathbf{y} = (y_1, y_2) = (7, 9)$. The squared distance of the competitive product from the desired levels is $\rho^2(\mathbf{max}, \mathbf{y}) = 10 = c$, whereas the squared distance of the product to be improved from the desired levels is $\rho^2(\mathbf{max}, \mathbf{z}) = 25$. Using equation (17), we get $a_1 = 7.53$, $a_2 = 8.0245$. These are the levels to which the current levels given by **z** should be shifted. The cost of doing so is $P_1 a_1 + P_2 a_2 = 69.75$ of the corresponding currency. This is exactly what the Excel Solver returns. And $\rho^2(\mathbf{max}, \mathbf{a}) = c$ is satisfied.

4.2. Example 2

Let the setting be now: $\mathbf{max} = (\max_1, \max_2) = (15, 20)$, $\mathbf{P} = (P_1, P_2) = (4.6, 7.1)$, $\mathbf{z} = (z_1, z_2) = (12, 14.5)$, $\mathbf{y} = (y_1, y_2) = (13, 17)$. The squared distance of the competitive product from the desired levels is here $\rho^2(\mathbf{max}, \mathbf{y}) = 13 = c$, whereas the squared distance of the product to be improved from the desired levels is $\rho^2(\mathbf{max}, \mathbf{z}) = 39.25$. Using equation (17) again, we have $a_1 = 13.03951$, $a_2 = 16.974$. These are the levels to which the current levels given by **z** should be moved. The cost of doing so is $P_1 a_1 + P_2 a_2 = 180.497$ of the corresponding currency. This is not *exactly* what the Excel Solver returns. The software returns: $a_1 = 13.03991$, $a_2 = 16.97368$ at a cost of $P_1 a_1 + P_2 a_2 = 180.4967$. The software result is slighty different and a tiny bit cheaper, but the software result gives, regarding condition (6), $\rho^2(\mathbf{max}, \mathbf{a}) = 13.00056$, whereas our solution given by (17) returns $\rho^2(\mathbf{max}, \mathbf{a}) = 13$ exactly. We can see the software finds an approximate solution only, which in this case is not feasible exactly, as opposed to solution (17) which is the exact feasible and optimal solution.

CONCLUSION

Since companies need to upgrade their products to stay competitive, questions arise how to change the product features to catch up with the competition, and how to do it cheaply. We presented a solution to this problem, using the mathematical concept of distance and optimization theory which give a straightforward solution in the form of an explicit equation. The virtue of our approach is such that not only can companies use this solution to optimize their product profile, but they can do so cheaply and *exactly*, as opposed to what they are provided with if they use an optimization software instead. As we demostrated in an example, optimization software may help solve the problem only approximately, which may not be enough. We stress that the problems we dealt with were of quantitative nature. If this is not the case, other approaches must be taken to make the product improvement, which are more focused on the overall quality of the product rather than its individual quantified features [4]. This problem, however, hardly ever happens. Also, our technique relies heavily on the proper measurement of product features, and this has to be taken care of, as well [6].

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