GENERALISED APPROACH TO DEVIATION ANALYSIS OF NON-LINEAR COMPANY DISCOUNTED VALUE MEASURE

Dana DLUHOŠOVÁ, Zdeněk ZMEŠKAL

VSB - Technical University of Ostrava, Faculty of Economics, Department of Finance, Ostrava, Czech Reublic, EU, dana.dluhosova@vsb.cz, zdenek.zmeskal@vsb.cz

Abstract

There is in the paper investigated problem of the deviation analysis application in decomposition of non-linear company value measure on the two-phase discounted method basis. Particular methods of the deviation analysis are described: gradual change method, method of deviation with residuals, logarithmic method, functional method, integrated method. Methods of company valuation are described. Illustrative example of the company value deviation analysis by the integral method is presented.

Keywords: deviation analysis, two-phase discounted valuation method, integrand method

1. INTRODUCTION

Analysis and methods of analysis are crucial methodological approach in financial management and decision-making. Sensitivity analysis and risk analysis is an important part of these analysis. Prediction and deviation analysis of economic system is key task for managing authorities.

One of the key tasks of the financial analysts is to analyse changes in basic ratios and deeply analyse factors affecting their changes the most. On the basis of the results it is possible to make some future decisions and actions. It is always possible to calculate initial (basic) value and comparative value. Methods for analysis of deviations are used in in-depth analysis of the past evolution of a given economic entity when comparing differences and their sources of these differences. It is possible to analyse in planning process the prediction and impact of deviation from the base scenario. In generalisation, it is possible to make all these in time-space region. Possible variants are shown in **Table 1**.

Table 1 Alternatives of analysis of deviation

Dimension comparison	Time phase	
	Past	Future
Time	A	D
Space	B	E
Time & Space	C	F

In finance, wide range of ratios is analysed, for example financial performance, profitability, solvency or liquidity. Important tool of the decision-making and the valuation is the value calculated two-phase yield method. The objective of the paper is to describe possible approaches in deviation analysis of the company value on the basic of the two-phase method. The integrand method is applied and verified.

2. METHODS OF ANALYSIS OF DEVIATIONS

Methods of deviation analysis are presented e.g. [6], [7], [9], [12], [14], [17]. Basically, there are two fundamental approaches to analysis of the basic ratios by the set of component ratios: (a) set of component ratios characterizing selected financial performance without exact mathematical accuracy, (b) pyramidal set of ratios, which is mathematically derived in the way, that the decomposition can be describe by the set of exact mathematical formulas and relationships.

Assume function of the basic ratio x depended on component ratios a_i , $x = x(a_1, a_2, \ldots a_n)$. Then i tis possible the change (deviation) of the basic ratio Δy_x express as a sum of the influences of the component ratios Δx_{a_i} as follows,

$$\Delta y_x = \sum_i \Delta x_{a_i} \tag{1}$$

It is necessary to note that both absolute deviations can be analysed, $\Delta x_{absolute} = x_1 - x_0$ and relative deviation $\Delta x_{relative} = (x_1 - x_0)/x_0$.

Generally, the most frequent relationships between component ratios are: additive relationships, if $x = \sum_i a_i = a_1 + a_2 + \ldots + a_n$, multiplicative relationships, if $x = \prod_i a_i$, or exponential relationships, $x = a_1^{\prod_j a_j} = a_1^{a_2 \cdot a_3 \cdot a_4 \cdot \ldots \cdot a_n}$, or non-linear relationships.

Basic idea of applied methods is to express the deviation of the basic ratio by approximation of the increment $\Delta x'(a_1, a_2, a_3)$ according to the change of the basic ratio relative to changes in the component ratios as follows,

$$\Delta y_x = \frac{\Delta x'(a_1, a_2, a_3)}{\Delta x'} \Delta y_x . \tag{2}$$

Due to the fact, that the Taylor series expansion will be applied, its general formula can be expressed as follows.

$$\Delta f(F_1, F_2, \cdots, F_n) = \sum_j \frac{\partial f(\cdot)}{\partial F_j} \cdot \Delta F_j + \frac{1}{2} \sum_j \sum_k \frac{\partial^2 f(\cdot)}{\partial F_j \cdot \partial F_k} \cdot \Delta F_j \cdot \Delta F_k + \\ + \frac{1}{6} \sum_j \sum_k \sum_l \frac{\partial^3 f(\cdot)}{\partial F_j \cdot \partial F_k \cdot \partial F_l} \cdot \Delta F_j \cdot \Delta F_k \cdot \Delta F_l + \cdots .$$

For the variables it holds,

$$\Delta f(F_1, F_2, F_3) = \left(\frac{\partial f(\,)}{\partial F_1} \Delta F_1 + \frac{\partial f(\,)}{\partial F_2} \Delta F_2 + \frac{\partial f(\,)}{\partial F_3} \Delta F_3 \right) +$$

$$+ \frac{1}{2} \cdot \left(\begin{array}{l} 2 \cdot \frac{\partial f^2(\,)}{\partial F_1 \partial F_2} \Delta F_1 \Delta F_2 + 2 \cdot \frac{\partial f^2(\,)}{\partial F_1 \partial F_3} \Delta F_1 \Delta F_3 + 2 \cdot \frac{\partial f^2(\,)}{\partial F_2 \partial F_3} \Delta F_2 \Delta F_3 + \\ \frac{\partial f^2(\,)}{\partial F_1^2} \Delta F_1^2 + \frac{\partial f^2(\,)}{\partial F_2^2} \Delta F_2^2 + \frac{\partial f^2(\,)}{\partial F_3^2} \Delta F_3^2 \end{array} \right) +$$

$$+\frac{1}{6}\cdot\left(\begin{array}{l} 6\cdot\frac{\partial f^3(\;)}{\partial F_1\partial F_2\partial F_3}\Delta F_1\Delta F_2\Delta F_3+ \\ 6\cdot\frac{\partial f^3(\;)}{\partial F_1\partial F_2^2}\Delta F_1\Delta F_2^2+6\cdot\frac{\partial f^3(\;)}{\partial F_1^2\partial F_2}\Delta F_1^2\Delta F_2+6\cdot\frac{\partial f^3(\;)}{\partial F_1\partial F_3^2}\Delta F_1\Delta F_3^2+ \\ 6\cdot\frac{\partial f^3(\;)}{\partial F_1^2\partial F_3}\Delta F_1^2\Delta F_3+6\cdot\frac{\partial f^3(\;)}{\partial F_2\partial F_3^2}\Delta F_2\Delta F_3^2+6\cdot\frac{\partial f^3(\;)}{\partial F_2^2\partial F_3}\Delta F_2^2\Delta F_3+ \\ \frac{\partial f^3(\;)}{\partial F_1^3}\Delta F_1^3+\frac{\partial f^3(\;)}{\partial F_2^3}\Delta F_2^3+\frac{\partial f^3(\;)}{\partial F_3^3}\Delta F_3^3 \end{array}\right)+\ldots\ldots$$

2.1. Additive relationship

The simplest linear function is the additive relationship. For three factors is the approximation by the Taylor series expansion,

$$\Delta x'(a_1+a_2+a_3)=\frac{\partial x(\;)}{\partial a_1}\cdot\Delta a_1+\frac{\partial x(\;)}{\partial a_2}\cdot\Delta a_2+\frac{\partial x(\;)}{\partial a_3}\cdot\Delta a_3=\Delta a_1+\Delta a_2+\Delta a_3 .$$

Influence of the factors is as follows:

$$\Delta x_{a_i}=\frac{\Delta a_i}{\sum_i \Delta a_i}\cdot\Delta y_x , \tag{3}$$

where $\Delta a_i = a_{i,1} - a_{i,0}$, $a_{i,0}$, respectively $a_{i,1}$ is the value of the *i-th* factor in a given period.

2.2. Multiplicative relationship

Multiplicative relationship for three factors is as $x = a_1 \cdot a_2 \cdot a_3$. There are five methods applicable: (a) method of the gradual changes, (b) method of the decomposition with surplus, (c) logarithmic method, (d) functional method and (e) integral method.

In the first two and integral method are applied, it is assumed that if one factor changes, the others are unchanged. If fourth and fifth method is applied it is assumed, that all the factors can changes simultaneously.

2.2.1. Multiplicative relationship for the method of the gradual changes

Decomposition for three factors is $x = a_1 \cdot a_2 \cdot a_3$. It follows,

$$\Delta x'(a_1\cdot a_2\cdot a_3)=\left.\frac{\partial x(\;)}{\partial a_1}\right|_{a_{1,0}\cdot a_{2,0}\cdot a_{3,0}}\cdot\Delta a_1+\left.\frac{\partial x(\;)}{\partial a_2}\right|_{a_{1,1}\cdot a_{2,0}\cdot a_{3,0}}\cdot\Delta a_2+\left.\frac{\partial x(\;)}{\partial a_3}\right|_{a_{1,1}\cdot a_{2,1}\cdot a_{3,0}}\cdot\Delta a_3=$$

$$=\Delta a_1\cdot a_{2,0}\cdot a_{3,0}+a_{1,1}\cdot\Delta a_2\cdot a_{3,0}+a_{1,1}\cdot a_{2,1}\cdot\Delta a_3$$

The influences are generally quantified without surplus as follows:

$$\Delta x_{a_1}=\Delta a_1\cdot a_{2,0}\cdot a_{3,0}\cdot\frac{\Delta y_x}{\Delta x},\Delta x_{a_2}=a_{1,1}\cdot\Delta a_2\cdot a_{3,0}\cdot\frac{\Delta y_x}{\Delta x},\Delta x_{a_n}=a_{1,1}\cdot a_{2,1}\cdot\Delta a_3\cdot\frac{\Delta y_x}{\Delta x}.$$

Generally it holds: $\Delta x_{a_i}=\Delta a_i\cdot\prod_{j<i}a_{j,1}\cdot\prod_{j>i}a_{j,0}\cdot\frac{\Delta y_x}{\Delta x}.$ (4)

2.2.2. Multiplicative relation of decomposition with remain

Influences are calculated with remain *R*, which is result of simultaneous changes several indices. So, particular influences are following,

$$\Delta x_{a_1} = \left(\Delta a_1 \cdot a_{2,0} \cdot a_{3,0} + R_1\right) \cdot \frac{\Delta y_x}{\Delta x}, \quad \Delta x_{a_2} = \left(a_{1,0} \cdot \Delta a_2 \cdot a_{3,0} + R_2\right) \cdot \frac{\Delta y_x}{\Delta x}, \quad \Delta x_{a_3} = \left(a_{1,0} \cdot a_{2,0} \cdot \Delta a_3 + R_3\right) \cdot \frac{\Delta y_x}{\Delta x}.$$

In general

$$\Delta x_{a_i} = \left(\Delta a_i \cdot \prod_{j \neq i} a_{j,0} + R_i\right) \cdot \frac{\Delta y_x}{\Delta x}. \tag{5}$$

2.2.3. Multiplicative relation of the logarithmic method

Derivation stem from ratios of indices,

$$I_x \equiv \frac{x_1}{x_0} = \frac{a_{1,1}}{a_{1,0}} \cdot \frac{a_{2,1}}{a_{2,0}} \cdot \frac{a_{3,1}}{a_{3,0}} = I_{a_1} \cdot I_{a_2} \cdot I_{a_3}.$$

So, margin is to be expressed, $\Delta x'(a_1 \cdot a_2 \cdot a_3) = \ln I_{a_1} + \ln I_{a_2} + \ln I_{a_3}$. Particular influences are calculated,

$$\Delta x_{a_i} = \frac{\ln I_{a_i}}{\ln I_x} \cdot \Delta y_x. \tag{6}$$

It is certain, that the continuous return is applied, since ratio logarithm is actually continuous return.

2.2.4. Multiplicative relation of functional method

All levels of Taylor expansion are applied,

$$\begin{aligned} \Delta x'(a_1 \cdot a_2 \cdot a_3) = {} & a_{2,0} \cdot a_{3,0} \cdot \Delta a_1 + a_{1,0} \cdot a_{3,0} \cdot \Delta a_2 + a_{1,0} \cdot a_{2,0} \cdot \Delta a_3 + \\ & + \frac{1}{2} \cdot \left(2 \cdot a_{3,0} \cdot \Delta a_1 \cdot \Delta a_2 + 2 \cdot a_{2,0} \cdot \Delta a_1 \cdot \Delta a_3 + 2 \cdot a_{1,0} \cdot \Delta a_2 \cdot \Delta a_3\right) \\ & + \frac{1}{6} \cdot 6 \cdot \Delta a_1 \cdot \Delta a_2 \cdot \Delta a_3. \end{aligned}$$

dividing previous formula by x_0, we get

$$\begin{aligned} \frac{\Delta x'(a_1 \cdot a_2 \cdot a_3)}{x_0} = {} & \frac{\Delta a_1}{a_{1,0}} + \frac{\Delta a_2}{a_{2,0}} + \frac{\Delta a_3}{a_{3,0}} + \\ & + \frac{1}{2} \cdot \left(2 \cdot \frac{\Delta a_1 \cdot \Delta a_2}{a_{1,0} \cdot a_{2,0}} + 2 \cdot \frac{\Delta a_1 \cdot \Delta a_3}{a_{1,0} \cdot a_{3,0}} + 2 \cdot \frac{\Delta a_2 \cdot \Delta a_3}{a_{2,0} \cdot a_{3,0}}\right) \\ & + \frac{1}{6} \cdot 6 \cdot \frac{\Delta a_1 \cdot \Delta a_2 \cdot \Delta a_3}{a_{1,0} \cdot a_{2,0} \cdot a_{3,0}}. \end{aligned}$$

Furthermore, equation is arranged as follows,

$$\begin{aligned} \frac{\Delta x'(a_1 \cdot a_2 \cdot a_3)}{x_0} = {} & \frac{\Delta a_1}{a_{1,0}} + \frac{\Delta a_2}{a_{2,0}} + \frac{\Delta a_3}{a_{3,0}} + 2 \cdot \left(\frac{1}{2} \cdot \frac{\Delta a_1 \cdot \Delta a_2}{a_{1,0} \cdot a_{2,0}}\right) + 2 \cdot \left(\frac{1}{2} \cdot \frac{\Delta a_1 \cdot \Delta a_3}{a_{1,0} \cdot a_{3,0}}\right) \\ & + 2 \cdot \left(\frac{1}{2} \cdot \frac{\Delta a_2 \cdot \Delta a_3}{a_{2,0} \cdot a_{3,0}}\right) + 3 \cdot \left(\frac{1}{3} \cdot \frac{\Delta a_1 \cdot \Delta a_2 \cdot \Delta a_3}{a_{1,0} \cdot a_{2,0} \cdot a_{3,0}}\right), \end{aligned}$$

Substituting $\Delta y_x = \dfrac{\Delta x'(a_1, a_2, a_3)}{x_0} \dfrac{x_0}{\Delta x'} \Delta y_x$ we find, that

$$\Delta y_x = \left(\frac{\Delta a_1}{a_{1,0}} + \frac{\Delta a_2}{a_{2,0}} + \frac{\Delta a_3}{a_{3,0}} + 2 \cdot \left(\frac{1}{2} \cdot \frac{\Delta a_1 \cdot \Delta a_2}{a_{1,0} \cdot a_{2,0}} \right) + 2 \cdot \left(\frac{1}{2} \cdot \frac{\Delta a_1 \cdot \Delta a_3}{a_{1,0} \cdot a_{3,0}} \right) + 2 \cdot \left(\frac{1}{2} \cdot \frac{\Delta a_2 \cdot \Delta a_3}{a_{2,0} \cdot a_{3,0}} \right) + \right.$$
$$\left. + 3 \cdot \left(\frac{1}{3} \cdot \frac{\Delta a_1 \cdot \Delta a_2 \cdot \Delta a_3}{a_{1,0} \cdot a_{2,0} \cdot a_{3,0}} \right) \right) \cdot \frac{x_0}{\Delta x} \cdot \Delta y_x .$$

It is known in finance, those formulas $R_{a_j} = \dfrac{\Delta a_j}{a_{j,0}}$ and $R_x = \dfrac{\Delta x}{x_0}$, means discrete returns. Then,

$$\Delta y_x = \left(R_{a_1} + R_{a_2} + R_{a_3} + 2 \cdot \left(\frac{1}{2} \cdot R_{a_1} \cdot R_{a_2} \right) + 2 \cdot \left(\frac{1}{2} \cdot R_{a_1} \cdot R_{a_3} \right) + 2 \cdot \left(\frac{1}{2} \cdot R_{a_2} \cdot R_{a_3} \right) + \right.$$
$$\left. 3 \cdot \left(\frac{1}{3} \cdot R_{a_1} \cdot R_{a_2} \cdot R_{a_3} \right) \right) \cdot \frac{1}{R_x} \cdot \Delta y_x .$$

Using previous formula, particular influences assigned to factors are,

$$\Delta x_{a_1} = \frac{1}{R_x} \cdot R_{a_1} \cdot \left(1 + \frac{1}{2} \cdot R_{a_2} + \frac{1}{2} \cdot R_{a_3} + \frac{1}{3} \cdot R_{a_2} \cdot R_{a_3} \right) \cdot \Delta y_x ,$$
$$\Delta x_{a_2} = \frac{1}{R_x} \cdot R_{a_2} \cdot \left(1 + \frac{1}{2} \cdot R_{a_1} + \frac{1}{2} \cdot R_{a_3} + \frac{1}{3} \cdot R_{a_2} \cdot R_{a_3} \right) \cdot \Delta y_x , \qquad (7)$$
$$\Delta x_{a_3} = \frac{1}{R_x} \cdot R_{a_3} \cdot \left(1 + \frac{1}{2} \cdot R_{a_1} + \frac{1}{2} \cdot R_{a_2} + \frac{1}{3} \cdot R_{a_2} \cdot R_{a_3} \right) \cdot \Delta y_x .$$

Similarly, formulas of more than three factors are to be derived.

2.2.5. Multiplicative relation of integrand method

Procedure of the integrand method is analogical to the functional method. Difference consist in applying only linear components of the Taylor expansion,

$\Delta x'(a_{1,0}, a_{2,0}, a_{3,0}) = a_{2,0} \cdot a_{3,0} \cdot \Delta a_1 + a_{1,0} \cdot a_{3,0} \cdot \Delta a_2 + a_{1,0} \cdot a_{2,0} \cdot \Delta a_3$, so

$\dfrac{\Delta x'}{x_0}(a_{1,0}, a_{2,0}, a_{3,0}) = \dfrac{\Delta a_1}{a_{1,0}} + \dfrac{\Delta a_2}{a_{2,0}} + \dfrac{\Delta a_3}{a_{3,0}}$. Substituting to $\Delta y_x = \dfrac{\Delta a_1}{a_{1,0}} \cdot \dfrac{\Delta a_2}{a_{2,0}} \cdot \dfrac{\Delta a_3}{a_{3,0}} \cdot \dfrac{x_0}{\Delta x'} \cdot \Delta y_x$, for any three factors,

$R_{a_j} = \dfrac{\Delta a_j}{a_{j,0}}$, a $R_{x'} = \dfrac{\Delta x'}{x_0}$, then $\Delta y_x = \left(R_{a_1} + R_{a_2} + R_{a_3} \right) \cdot \dfrac{1}{R_{x'}} \cdot \Delta y_x$.

Particular influences should be given subsequently,

$$\Delta x_{a_1} = \frac{R_{a_1}}{R_{x'}} \cdot \Delta y_x , \quad \Delta x_{a_2} = \frac{R_{a_2}}{R_{x'}} \cdot \Delta y_x , \quad \Delta x_{a_3} = \frac{R_{a_3}}{R_{x'}} \cdot \Delta y_x .$$

It is apparent, that whatever number of elements influences calculation is as follows,

$$\Delta x_{a_j} = \frac{R_{a_j}}{R_{x'}} \cdot \Delta y_x , R_{x'} = \sum_{j=1}^{N} R_{a_j} . \qquad (8)$$

2.3. Exponential relation of the integrand method

In coincidence with previous procedure, the function will be analysed, $x = a_1^{\prod_{i=2} a_i}$. Then, due to linear Taylor expansion, $\Delta x' = \frac{\partial x(\ )}{\partial a_1} \cdot \Delta a_1 + \sum_{i=2} \frac{\partial x(\ )}{\partial a_i} \cdot \Delta a_i = \prod_{i=2} a_i \cdot a_1^{\left(\prod_{i=2} a_i - 1\right)} \cdot \Delta a_1 + \sum_{i=2} \ln a_1 \cdot a_1^{a_i} \cdot \Delta a_i$.

It imply, that

$$\Delta x_{a_1} = \frac{\prod_{i=2} a_i \cdot a_1^{\left(\prod_{i=2} a_i - 1\right)} \cdot \Delta a_1}{\Delta x'} \cdot \Delta y_x \text{, and } \quad \Delta x_{a_i} = \frac{\ln a_1 \cdot a_1^{a_i} \cdot \Delta a_i}{\Delta x'} \cdot \Delta y_x \text{ , for } i \geq 2 . \tag{9}$$

2.4. Evaluation and summary

Commonly applicable methods for non-linear functions are the functional method and the integrand method. Advantages of the gradual changes are simplicity and no remain. Non-comfortable is fact that influence value depends on the factors ordering. Pros of the decomposition with remain is results independence on ordering. Method cons consist in non-uniqueness of remain distribution. Applying the logarithmic method the simultaneous changes are reflected. On the other side, existence of negative ratio not allows to apply the method. The functional method is based on the discrete returns. Advantage coincide with logarithmic method, constraint of the negative ratio does not exist. Advantages of the integrand method are similar to the functional method. Interpretation and calculation of the deviation should be more simple including the signs and direction.

3. DECOMPOSITION OF COMPANY VALUE DUE TO TWO-PHASE DISCOUNTED METHOD

Approaches to valuation are presented e. g. see [1], [2], [3], [4], [5], [8], [10], [11], [13], [15], [16]. Two-phase discounted valuation method *V* is often applied valuation method. The value computation is formulated as follows,

$$V = V1 + V2 = \sum_{t=1}^{T} FCF_t \cdot (1+R)^{-t} + \frac{FCF_{T+1}}{R} (1+R)^{-T} \text{ ,}$$

where FCF_t are free financial flows in the year t , R is cost of capital, T is the first period, length.

3.1. Deviation decomposition of company due to integrand method

The linear part of Taylor expansion is in integrand method applied.

$$\Delta V' = \sum_{t=1}^{T} \frac{\partial V(\ )}{\partial FCF_t} \Delta FCF_t + \frac{\partial V(\ )}{\partial FCF_{T+1}} \Delta FCF_{T+1} + \frac{\partial V(\ )}{\partial R} \Delta R \text{ , so}$$

$$\Delta V' = \sum_{t=1}^{T} (1+R)^{-t} \cdot \Delta FCF_t + R^{-1} (1+R)^{-T} \cdot \Delta FCF_{T+1} + \sum_{t=1}^{T} FCF_t \cdot (-t)(1+R)^{-t-1} \cdot \Delta R +$$
$$FCF_{T+1} \cdot \frac{-\left[\sum_{k-0}^{T} \binom{T}{k} \cdot (k+1) R^k\right]}{\left[\sum_{k-0}^{T} \binom{T}{k} \cdot R^{k+1}\right]^2} \cdot \Delta R \qquad .$$

The last component is derived as follows, $\frac{\partial\left[R^{-1}(1+R)^{-T}\cdot\Delta FCF_{T+1}\right]}{\partial R}\Delta R = FCF_{T+1}\cdot\frac{-\left[R(1+R)^{T}\right]'}{\left[R(1+R)^{T}\right]^{2}}\cdot\Delta R =$

$$= FCF_{T+1}\cdot\frac{-\left[\sum_{k-0}^{T}\binom{T}{k}\cdot R^{k+1}\right]'}{\left[\sum_{k-0}^{T}\binom{T}{k}\cdot R^{k+1}\right]^{2}}\cdot\Delta R = FCF_{T+1}\cdot\frac{-\left[\sum_{k-0}^{T}\binom{T}{k}\cdot(k+1)\cdot R^{k}\right]}{\left[\sum_{k-0}^{T}\binom{T}{k}\cdot R^{k+1}\right]^{2}}\cdot\Delta R$$

Influences of particular factors are following,

$$\Delta x_{FCF_t} = \frac{(1+R)^{-t}\cdot\Delta FCF_t}{\Delta V'}\cdot\Delta y_x\ ,$$

$$\Delta x_{FCF_{4T+1}} = \frac{R^{-1}(1+R)^{-T}\cdot\Delta FCF_{T+1}}{\Delta V'}\cdot\Delta y_x$$

$$\Delta x_R = \frac{\sum_{t=0}^{T}FCF_t\cdot(-t)(1+R)^{-t-1}\cdot\Delta R}{\Delta V'}\cdot\Delta y_x + FCF_{T+1}\cdot\frac{-\left[\sum_{k-0}^{T}\binom{T}{k}\cdot(k+1)R^{k}\right]}{\left[\sum_{k-0}^{T}\binom{T}{k}\cdot R^{k+1}\right]^{2}}\cdot\Delta R\cdot\Delta y_x\ , \tag{10}$$

3.2. Decomposition of value deviation for 3 periods of the first phase

Particular integrand method for three periods of the first phase,

$$\Delta V' = \sum_{t=1}^{T=3}\frac{\partial V(\ )}{\partial FCF_t}\Delta FCF_t + \frac{\partial V(\ )}{\partial FCF_{T+1}}\Delta FCF_{T+1} + \frac{\partial V(\ )}{\partial R}\Delta R\text{ , so}$$

$$\Delta V' = \sum_{t=1}^{T=3}(1+R)^{-t}\cdot\Delta FCF_t + R^{-1}(1+R)^{-3}\cdot\Delta FCF_{3+1} + \sum_{t=1}^{T=3}FCF_t\cdot(-t)(1+R)^{-t-1}\cdot\Delta R +$$

$$\frac{-\left[\sum_{k-0}^{T=3}\binom{T}{k}\cdot(k+1)R^{k}\right]}{\left[\sum_{k-0}^{T=3}\binom{T}{k}\cdot R^{k+1}\right]^{2}}\cdot\Delta R\cdot .$$

Influences of particular factors are following,

$$\Delta x_{FCF_t} = \frac{(1+R)^{-t}\cdot\Delta FCF_t}{\Delta V'}\cdot\Delta y_x\ ,$$

$$\Delta x_{FCF_{T+1}} = \frac{R^{-1}(1+R)^{-T}\cdot\Delta FCF_{T+1}}{\Delta V'}\cdot\Delta y_x$$

$$\Delta x_R = \frac{\sum_{t=1}^{T=3} FCF_t \cdot (-t)(1+R)^{-t-1} \cdot \Delta R}{\Delta V'} \cdot \Delta y_x + + FCF_{T+1} \cdot \frac{-\left[1+12 \cdot R^1 + 9 \cdot R^2 + 4 \cdot R^3\right]}{\left[R + 6 \cdot R^2 + 3 \cdot R^3 + R^4\right]^2} \cdot \Delta R \cdot \Delta y_x \tag{11}$$

4. ILLUSTRATIVE EXAMPLE OF THE INTEGRAND METHOD APPLICATION FOR TWO-PHASE VALUATION METHOD

Deviation analysis of the company value is frequent problem. One of the company value determinations is two-phase discounted value method. By this way, various alternatives should be compared, e.g. forecast and reality (post audit), or scenarios with basis alternative.

4.1. Introduction

Input data of basis and comparison alternatives company value are given. The deviation value of company value is 83.1 monetary unit. The task is to identify particular influences causing the deviation. The necessary input data, free cash-flow *FCF*, cost of capital *R*, value of the first phase *V1* value of the second phase *V2*, total value *V*, are presented in **Table 2**.

Table 2 Input data *V* - deviation

Alternative	Indices					Value		
	FCF_1	FCF_2	FCF_3	FCF_4	*R*	*V1*	*V2*	*V*
Basis (0)	100	280	300	310	10.0%	570.2	2562.0	3132.2
Comparable (1)	90	250	290	300	9.0%	528.1	2687.3	3215.4
Deviation	-10	-30	-10	-10	-1.0%	-42.1	125.3	83.1

4.2. Solution

Deviations are calculated applying integrand method is calculated due to (11). Results are presented in **Table 3**. We can see that total deviation of *V* is 83.1 m. u., influence of free cash flow change is negative, of the cost of capital positive equal 151.4. Influence of the first phase is -17.8 and the second phase 101 m. u.

Table 3 Results of two-phase discounted value *V* deviation

Item	Indices					Value		
	FCF_1	FCF_2	FCF_3	FCF_4	*R*	*V1*	*V2*	*V*
$\Delta V'_i$ components	-9.1	-24.8	-8.3	-82.6	276.9	-32.6	184.7	152.1
Influence (%)	-6.0	-16.3	-5.4	-54.3	182.1	-21.4	121.4	100.0
Influence (m. u.)	-5.0	-13.6	-4.5	-45.2	151.4	-17.8	101.0	83.1

CONCLUSION

The intention of the paper was deviation analysis of company value on the two phase discounted value method. Non-linear value function is analysed by the integrand method. The illustrative example of deviation analysis was investigated. We can say and state that integrand method is suitable method for value deviation analysis.

ACKNOWLEDGEMENT

This paper has been elaborated in the framework of the IT4Innovations Centre of Excellence project, reg. no. CZ.1.05/1.1.00/02.0070 supported by Operational Programme 'Research and Development for Innovations' funded by Structural Funds of the European Union and state budget of the Czech Republic. The research was supported by the European Social Fund within the project CZ.1.07/2.3.00/20.0296 as well.

REFERENCES

[1] BREALEY, R.A., MYERS, C.M., ALLEN, F. *Principles of corporate finance*, 11th ed. New York: McGraw-Hill, 2005.

[2] COPELAND, T.E, WESTON, J.F., SHASTRI, K. *Financial theory and corporate policy*. 4th ed. Harlow: Pearson, 2005.

[3] ČULÍK, M. Real option application for modular project valuation. In *24th International Conference on Mathematical Methods in Economics*, 2006, pp. 123-130.

[4] DAMODARAN, A. Damodaran on valuation, security analysis for investment and corporate finance. Chichester: Wiley, 1994.

[5] DIXIT, A.K., PINDYCK, R.S. *Investment under Uncertainty*. Princeton University Press, 1994.

[6] DLUHOŠOVÁ, D. Evolution and approaches to firm's and industry's performance analysis in transition and reconstruction phase of economy development. *ECON*, 2003, vol. 10, pp. 69-85.

[7] DLUHOŠOVÁ, D. a kol. *Nové přístupy a finanční nástroje ve finančním rozhodování*. Ostrava: VSB-TU Ostrava, Ekonomická fakulta, 2004.

[8] DLUHOŠOVÁ, D., RICHTAROVÁ, D., VALECKÝ, J., ZMEŠKAL, Z. *Finanční řízení a rozhodování podniku.* 3. vyd. Praha: Ekopress, 2010.

[9] DLUHOŠOVÁ, D. Přístupy k analýze finanční výkonnosti firem a odvětví na bázi metody EVA - Economic Value Added. *Finance a úvěr- Czech Journal of Economics and Finance,* vol. 54, no. 11-12, 2004.

[10] KRAĽOVIČ, J., VLACHYNSKÝ, K. *Finančný manažement*. Bratislava: Edícia Ekonomia, 2002.

[11] VALACH, J. Investiční rozhodování a dlouhodobé financování. Praha: Ekopress, 2001.

[12] ZALAI, K. a kol. *Finančno-ekonomická analýza podniku*. Bratislava: Sprint, 2000.

[13] ZMEŠKAL, Z. Dynamický optimalizační model volby odpisové metody, tvorby a užití finančních zdrojů. *Finance a úvěr - Czech journal of economics and finance,* vol. 45, no. 1, 1995, pp. 29-36.

[14] ZMEŠKAL, Z. a kol. *Finanční modely*. 3. upravené vydání. Praha: Ekopress, 2013.

[15] ZMEŠKAL, Z. Application of the American Real Flexible Switch Options Methodology A Generalized Approach. *Finance a Úvěr - Czech Journal of Economics and Finance,* vol. 58, 2008, pp. 261-275.

[16] ZMEŠKAL, Z. Application of the Fuzzy - Stochastic Methodology to Appraising the Firm Value as a European Call Option. *European Journal of Operational Research,* vol. 135, no. 2, 2001, pp. 303-310.

[17] ZMEŠKAL, Z., DLUHOŠOVÁ, D. Metody analýzy odchylek ukazatele present value - zobecněný přístup. Conference proceedings, Finanční řízení podniků a finančních institucí. Ostrava, 2012.