THE SOLUTION OF THE ALLOCATION PROBLEM USING DYNAMIC PROGRAMMING

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Abstract

One of the main problems companies have to solve is the distribution of resources among the potential beneficiaries. In many cases, this problem can be presented as a linear programming task with a linear objective function and constraining conditions. However, in some cases, the formulation of the problem as a mathematical programming task leads to integral or non-linear models requiring difficult or costly solution procedures. Dynamic programming offers a better way of dealing with these complex examples. The aim of the article is to show a suitable utilization of dynamic programming using an example of an allocation task.

Keywords: dynamic programming, optimization, allocation problem

1. INTRODUCTION

Managerial decision-making issues solved by linear programming methods usually involve simple decisions. However, the manager often takes into account the sequence of decisions, where each decision affects the next one. A tool that allows us to deal with such types of decision-making issues is called dynamic programming.

There is no easy model how to solve managerial problems by means of dynamic programming. That is why these issues are put into groups, where each of them has its own formulation, manner and solution method. The basic approach, method and logic of problem solving by means of dynamic programming is the same.

2. THE ISSUE OF ALLOCATION DECISION-MAKING

The allocation of resources among the potential beneficiaries is one of the main problems of today's enterprises [1]. In decision-making cases with linear objective function and constraining conditions, it is a decision-making issue belonging to the category of linear programming [2]. In many cases, however, the formulation of mathematical programming leads to integral or linear models, the subsequent solution of which is difficult [3]. Dynamic programming offers a better way of solving some of these complex cases.

3. EXAMPLE OF INVESTMENT DECISION-MAKING

The management of a company is considering allocating 400 million crowns among three companies. The decision on the allocation of 0, 100, 200, 300 or 400 million into one company has already been taken (the investment is in the order of 100 million dollars). Each of the companies has presented its prospect of the annual rate of return, which corresponds to different levels of invested funds that are listed in **Table 1**. The problem is to determine the optimal allocation of funds in each of the companies in such a way to maximize the anticipated total annual rate of return. This problem cannot be solved by linear programming, because it is a problem of integral programming, and the individual rates of returns are not linear.

The decision-making issue is divided into three stages, where each stage represents allocation in one company. We will understand it as a sequence of decisions on the individual sub-problems. The individual relationships among the stages are shown in **Fig. 1**.

Table 1 The individual investment options

Invested amounts (mil. CZK)	Annual recovery of investment (mil. CZK)		
	Company A	Company B	Company C
0	0	0	0
100	20	30	40
200	50	60	90
300	150	120	110
400	140	150	160

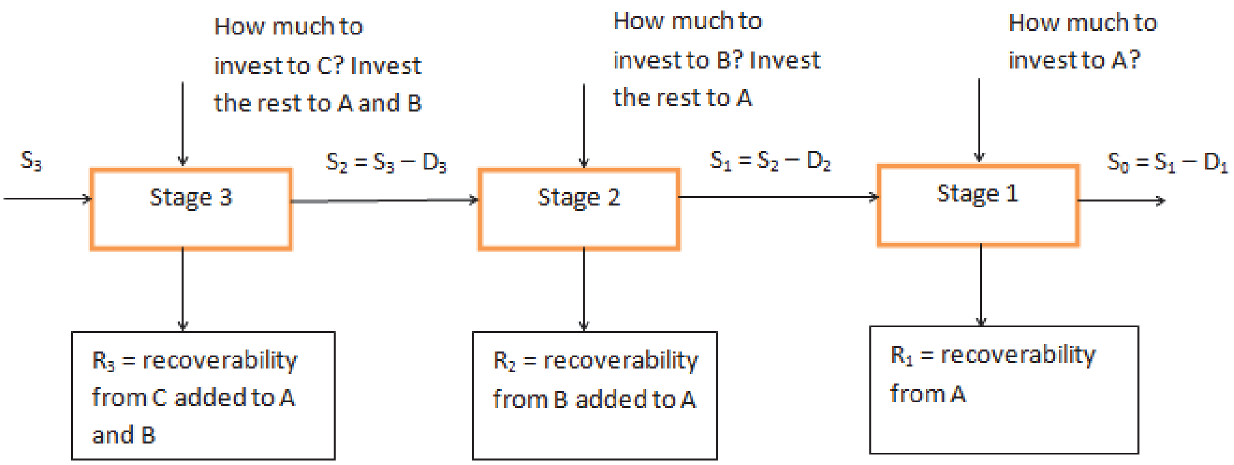

Fig. 1 The allocation decision-making issue divided into stages

A reverse approach will be applied here again. First, we allocate the funds to company A (it is considered to be the last company), following by B and by C company.

There may be 5 different states in each stage; you can allocate 0, 100, 200, 300 or 400 million CZK.

4. SOLUTION

0, 100, 200, 300 or 400 million CZK, marked as S_1, is available to be allocated in company A during **the first stage**. The calculated recovery of investment in company A is presented in **Table 2**.

When 0, 100, 200 or 300 million CZK is available, the best solution is to allocate all the money. However, when 400 million CZK is available, the best solution is to allocate only 300 million CZK, because the optimal recovery of 300 million CZK is higher than the recovery of 400 million CZK. (this is unusual, but possible). The last column of **Table 2** presents the highest yield (optimal) in each line. The numbers printed in bold are the highest ones and they show the optimal decision.

The second stage should determine how to allocate the available millions of CZK between A and B. Let us designate the amount that is available for allocation in A and B as S_2. From the allocated amount S_2, company B receives D_2, while the rest of S_2 - D_2 = S_1 is available for allocation in company A, using the best way calculated in the first stage.

There are several allocation options for each value of S_2 (0, 100, 200, 300 and 400 million CZK), and all of them must be taken into account. All five possible states will be examined.

Table 2 The first investment stage into company A

S_1 available for company A	Decision D_1 - how much to invest to company A with the state					Optimal yield
	0	100	200	300	400	
0	**0**					**0**
100	0	**20**				**20**
200	0	20	**50**			**50**
300	0	20	50	**150**		**150**
400	0	20	50	150	**140**	**150**

For the state of S_2 = 0 - no investment equals no recoverability

For the state of S_2 = 100 - either 100 in B and 0 to A - total recoverability of 30 + 0 = 30 or 0 to B and 100 to A - the total recoverability of 0 + 20 = 20. It is clear that 100 to B is a better investment. If we have 100 million CZK left to be invested into A and B, then it goes to B. This information then enters **Table 3**, which includes the calculations for all the remaining states. The optimal yield is calculated in each state.

The calculations in the table represent the total yield, which is a sum of immediate yields + optimal yield from the first stage.

S_2 = 3, where the line for **Table 3** was calculated as shown in **Table 4**, where the total yields are calculated for each line, the largest of which is selected and indicated as the optimal total yield.

Table 3 Calculation for S_2

S_2 usable for companies A and B	Decision D_2 - how much to allocate to B, the rest to A in optimal rate					Optimal total yield
	0	100	200	300	400	
0	**0**					**0**
100	0+20=20	30+0=**30**				**30**
200	0+50=50	30+20=50	60+0=**60**			**60**
300	0+150=**150**	30+50=80	60+20=80	120+0=120		**150**
400	0+150=150	30+150=**180**	60+50=110	120+20=140	150+0=150	**180**

Table 4 Detailed calculation S_2

D_2 investment to B	The rest and allocation to A	Immediate yield	Optimum from the 1st stage	Total yield
0	**300**	**0**	**150**	**150**
100	200	30	50	80
200	100	60	20	80
300	0	120	0	120

Optimal total yield

The investment decision-making for company C is made during **the third stage** and the remaining amount is then allocated between A and B using the best way, according to the procedure described in the second stage. Only the state of S_3 = 4 will be presented in the last stage. The other states are worse, so they were dropped.

The calculations are shown in **Table 5** in a standard form used previously, and in a rather more detailed form in **Table 6**. The best allocation is:

D_3 = 100; D_2 = 0; D_1 = 300, i.e., 100 to C and 300 to A with the recoverability of 190 mil. CZK.

Table 5 Calculation for S_3 = 4

S_3 usable for companies A, B and C	Decision D_3 - how much to allocate to C					Optimal total yield
	0	100	200	300	400	
4	0+180=180	**40+150=190**	90+60=150	110+30=140	160+0=160	**190**

Table 6 Detailed calculation S_3 = 4

Options	Recoverability for C	Recoverability from the best allocation between A and B	Total yield	
D_3 = 4 to C 0 to A+B	160	0	160	
D_3 = 3 to C, 1 to A+B	110	30 (100 to B)	140	
D_3 =2 to C, 2 to A+B	90	60 (200 to B)	150	
D_3 = 1 to C, 3 to A+B	**40**	**150 (300 to B)**	**190**	**Optimal total**
D_3 = 0 to C, 4 to A+B	0	180 (100 to B, 300 to A)	180	

5. DISCUSSION OF RESULTS

This example can help us to make some valuable notes:

- Optimal recoverability is calculated for each value of S in each stage during the analysis,
- The recoverability for the given investment tactics, when S increases (decreases) in the rate of 100 million CZK, can be easily determined from the pre-calculated tables,
- Sensitivity analysis can be easily performed, when the management decides to consider only two companies, then the optimal solution can be found directly in the intermediate computations, e.g., if we are considering only company A and B, the best solution is calculated from **Table 3** as D_1 = 100 a D_1 = 300, the total recoverability of 180 million CZK,
- The dynamic programming procedure also identifies the second best option. In this case, it is **Table 5**, where D_3 = 0, i.e., nothing is allocated into company C, 100 to B and 300 to A, with an expected recoverability of 180 million CZK.
- Adding a new company to the issue only adds another stage to the calculation,
- Adding more money to invest only adds more states to the calculation.
- The dynamic programming process of this issue required 18 calculations. Solution by means of complete enumeration would require 15 calculations. Again, we are not going to make any savings on calculations with such small problems. However, if the problems were bigger, the savings on calculations would be considerable. [4]

CONCLUSION

Dynamic programming aims at finding the optimal solution of a decision-making issue that will break it down into smaller sub-problems identified as stages. There are several states or positions for each stage, for each sub-problem existing within a given problem.

The procedure of dynamic programming begins in the last stage, where a set of optimal solutions is given for each state within the stage (you can also use a linear programming algorithm). This set is then used for the next solution stage and the process continues until the original decision-making issue is solved.

Each approach has to be tailor-made for each different type of problem. The problem is always different, and therefore a new formulation must be designed. [5]

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