

**AN ANALYSIS OF THE  $f_c$  AND  $\epsilon_N$  PARAMETERS IN LOCAL GURSON-BASED MODELLING OF FAILURE OF S235JR STEEL AT HIGH STRESS TRIAXIALITY**

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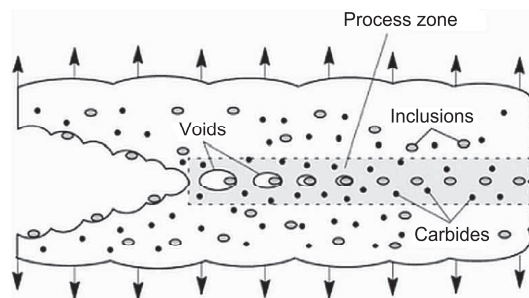
**Abstract**

The study deals with numerical modelling of the ductile fracture and failure of elements subjected to static tension in the spatial stress state. The analysis was conducted for notched specimens made of S235JR steel in the high stress triaxiality condition. The Gurson-based approach was used to study the microstructural processes contributing to the ductile fracture of S235JR steel. The Gurson-Tvergaard-Needleman (GTN) material model, able to calculate the relationships between the material microstructure and its strength and between the microstructural changes and plastic fracture, was applied to locally model the initiation of damage processes in the material structure. The analysis focuses on the effect of two of the most crucial GTN model parameters - the critical void volume fraction,  $f_c$  and mean strain for void nucleation,  $\epsilon_N$  - on the material response. Two different values of the  $f_c$  and  $\epsilon_N$  parameters were considered: those obtained experimentally and those assumed on the basis of the literature data. The simulation results were compared with the data obtained through the modelling of notched specimens under tensile loading, assuming that the GTN model of a porous material is used for the whole element.

**Keywords:** Gurson-Tvergaard-Needleman material model, GTN, numerical modelling, S235JR steel

**1. INTRODUCTION**

The main factor determining the strength of the material and the mechanisms of its failure is the microstructure. Failure processes of steel and other metallic materials are closely linked with microstructural defects in the form of voids, which are initiated at inclusions and precipitates present in the material. The void growth is the most significant step, which determines the failure process. It involves formation of the macroscopically planar fracture process zone. Its thickness corresponds to the distance between one or two voids. There are intense plastic flows between the voids in this region, while such phenomenon is not observed in the remaining area, where there is no significant voids growth also (see **Fig. 1**).



**Fig. 1** Microstructural model of ductile fracture, based on [1]

Processes, which occur in the material microstructure, result in a reduction of strength of material at the macro level. Generally, the strength of material, defined by normal stress  $\sigma$ , is reduced by the damage parameter  $d$ , which is a factor determining the impact of the development of microstructure defects on the prevailing state of stress. This reduced stress can be established as  $\sigma(1-d)$ .

These phenomena can be modeled using advanced material models based on damage mechanics. One of such models is the Gurson material model [2], which takes into account the impact of microstructure defects, defined by the void volume fraction  $f$ , on the strength of the material. The modification of the original Gurson condition is the Gurson-Tvergaard-Needleman material model, denoted as GTN [3, 4]. Currently, it is one of the fundamental damage material models used in the failure analysis of different metallic materials.

Taking into account the microstructural phenomena in the analytical solutions is complicated and requires a multidisciplinary approach. Numerical methods, such as finite element method (FEM), offer many new opportunities in this field, such as simulation of the material behavior and analysis of ongoing processes.

One of the methods of numerical analysis and simulation of microstructural processes is the local modeling of failure in metals basing on the Gurson-based material models for porous media. It consists in physically modeling of a layer containing a pre-existing population of similar sized voids in the numerical model. Thickness of this layer is established basing on the material microstructure. The Gurson-based material model (e.g. GTN) is applied in this layer for all computational cells, so that they contain a single void of some initial volume. The Gurson constitutive relation for dilatant plasticity defines the void growth. It results in reduction of stress carrying capacity leading to the material failure. For the remaining part of the element, the elastic-plastic material model is assumed. So damage material model is assumed for process zone only, according to the physical processes observed in metals.

Such an approach is the subject of the presented studies, where the failure processes that occur in S235JR steel subjected to tension are simulated. This is continuation and extension of research performed by Authors [5-10]. The main objective was to analyze and examine the effects which occur during local Gurson-based modelling of failure of S235JR steel at high stress triaxiality. The cylindrical notched tensile specimens are considered, where due to the presence of the notch the spatial stress state is observed. It is defined by the so-called stress triaxiality  $\sigma_m/\sigma_e$ , where  $\sigma_m$  is the hydrostatic stress and  $\sigma_e$  is the effective stress. The geometry of the elements analyzed defines high value of  $\sigma_m/\sigma_e = 0.739$ , which is significantly higher than those observed for the samples without the notch.

## 2. GURSON-TVERGAARD-NEEDLEMAN MATERIAL MODEL

Gurson-Tvergaard-Needleman (GTN) material model is the modification of the original Gurson condition taking into consideration the effect of microstructural defects on the material strength. The GTN yield condition developed in several studies [3, 4] is written in the form:

$$\Phi = \left( \frac{\sigma_e}{\sigma_0} \right)^2 + 2q_1 f^* \cosh \left( -q_2 \frac{3\sigma_m}{2\sigma_0} \right) - (1 + q_3 f^{*2}) = 0 \quad (1)$$

where:  $\sigma_e$  - effective stress,  $\sigma_0$  - yield stress,  $\sigma_m$  - hydrostatic stress,  $f^*$  - modified void volume fraction,  $q_i$  - Tvergaard's parameters,  $i = 1, 2, 3$ .

As can be seen from (1), the stress carrying capacity in the GTN material model is affected by two microstructural parameters, i.e. modified void volume fraction  $f^*$  and Tvergaard's parameters  $q_1$ .

The modified void volume fraction  $f^*$  is related initially to material porosity defined by initial void volume fraction  $f_0$ . Value of the  $f^*$  parameter is changed during the deformation according to the following function:

$$f^* = \begin{cases} f & \text{for } f \leq f_c \\ f_c + \frac{\bar{f}_F - f_c}{f_F - f_c} (f - f_c) & \text{for } f_c < f < f_F \\ \bar{f}_F & \text{for } f \geq f_F \end{cases} \quad (2)$$

where:  $f_c$  - critical void volume fraction,  $f_F$  - void volume fraction at final fracture,  $\bar{f}_F = (q_1 + \sqrt{q_1^2 - q_3})/q_3$ .

Depending on the current void volume related to total volume in material unit, the modified void volume fraction  $f^*$  takes different values. When the voids growth occurs, the  $f^*$  parameter is dependent on the two critical void volume fractions,  $f_c$  and  $f_F$ . The  $f_c$  parameter corresponds to the void volume fraction observed for beginning of the void coalescence process. On a macro level this is manifested by sudden drop of the  $\sigma(\epsilon)$  curve, determining the stress carrying capacity of the material.

The void growth rule for increase in the void volume fraction  $\dot{f}$  is the most often defined as:

$$\dot{f} = \dot{f}_{gr} + \dot{f}_{nucl} = (1-f) \dot{\epsilon}^{pl} : \mathbf{I} + \frac{f_N}{s_N \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\epsilon_{em}^{pl} - \epsilon_N}{s_N} \right)^2 \right] \cdot \dot{\epsilon}_{em}^{pl} \quad (3)$$

where:  $\dot{f}_{gr}$  - void growth rate,  $\dot{f}_{nucl}$  - void nucleation rate,  $f_N$  - volume fraction of void nucleating particles,  $s_N$  - standard deviation of nucleation strain,  $\dot{\epsilon}^{pl}$  - plastic strain rate tensor,  $\mathbf{I}$  - second-order unit tensor,  $\epsilon_N$  - mean strain for void nucleation,  $\epsilon_{em}^{pl}$  - effective strain,  $\dot{\epsilon}_{em}^{pl}$  - effective strain rate.

Mean strain for void nucleation  $\epsilon_N$  defines the strain level when first voids are formed in the material microstructure. Thus, the  $\epsilon_N$  parameter significantly determines the initiation of the failure process, resulting in the reduction of the stress carrying capacity of the material.

### 3. EXPERIMENTAL INVESTIGATIONS

In the first stage, the research program included the microstructural examinations and standard tensile tests of S235JR steel, necessary to obtain mechanical properties and GTN model parameters, and tensile tests in a spatial stress state.

#### 3.1 Microstructural examinations

S235JR is a low-carbon, non-alloy structural steel, characterized by a large amount of impurities. The chemical composition of tested material - maximum content of elements in [%] is: C = 0.14, Mn = 0.54, Si = 0.17, P = 0.016, S = 0.026, Cu = 0.29, Cr = 0.12, Ni = 0.12, Mo = 0.03, V = 0.002 and N = 0.01. During the microstructural examinations the structure and basic parameter such as material porosity was determined. A ferritic-perlitic structure with a large number of non-metallic inclusions was observed for S235JR steel. The initial void volume fraction,  $f_0$  was determined as a parameter characterized porosity of the material. It was determined as  $f_0 = 0.0017 = 0.17\%$  [5-7].

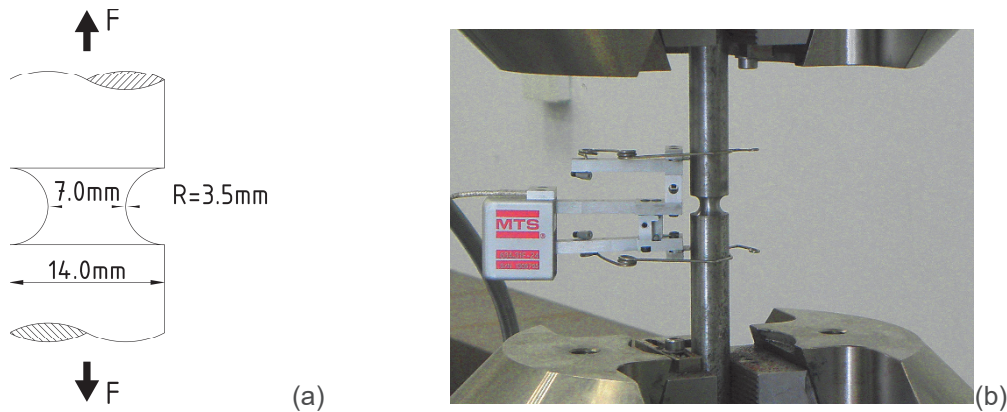
#### 3.2 Standard tensile tests

In order to describe mechanical properties of S235JR steel, standard static tensile tests were performed according to PN-EN10002-1:2004 [11]. The sample size was  $n = 8$  specimens with a circular cross-section of the nominal diameter  $d = 10$  mm and the initial gauge length  $l_0 = 50$  mm.

The basic strength parameters for S235JR steel for the significance level of 0.05 were obtained as follows: the proof strength  $R_{0.2} = 318.3 \pm 2.59$  MPa with standard deviation  $s = 3.73$  MPa, the tensile strength  $R_m = 457.4 \pm 4.91$  MPa with standard deviation  $s = 7.09$  MPa and the displacement percentage elongation  $A_t = 33.9 \pm 1.47\%$  with standard deviation  $s = 2.13\%$  [5, 6].

### 3.3 Tensile tests in spatial stress state

The analysis included experimental and numerical investigations of elements subjected to tension in spatial stress state. The tensile tests of cylindrical elements with notch were performed, as shown in **Fig. 2**. The geometry of elements allowed to analyze the spatial stress state for stress triaxiality  $\sigma_m/\sigma_e = 0.739$ .



**Fig. 2** Sample geometry (a) and tensile test in spatial stress state (b)

## 4. GTN PARAMETERS FOR S235JR STEEL

Apart from the basic mechanical properties, it was necessary to determine several microstructural parameters in order to apply the GTN material model in the analysis. The methods and results developed by Kossakowski [5, 6] were used to establish GTN parameters, which are listed below for S235JR steel.

The first of all, the stress-strain relation  $\sigma(\epsilon)$  was defined to describe the material behavior. For S235JR steel the elastic-plastic material model with hardening was used. The porosity of the material was taken into consideration in GTN material model by assuming the void volume fraction, which was determined during the microstructural analysis as  $f_0 = 0.0017$ . Basing on the Faleskog results [12], Tvergaard's parameters were determined on the relations of proof strength to the modulus of elasticity  $R_{0.2}/E = 0.00155$  and the strain-hardening exponent  $N = 0.183$  as  $q_1 = 1.90$ ,  $q_2 = 0.81$  and  $q_3 = 3.61$ .

The critical void volume fractions  $f_c$  were assumed in two ways, i.e. basing on the literature data and values determined during experiments. Taking into considerations the results of investigations [5, 6], critical parameter  $f_c$  was assumed as  $f_c = 0.06$ . In the second step the method developed by Kossakowski and Wciślik [7] was used. The notched samples shown in **Fig. 2** were used to determine experimentally the critical parameter  $f_c$ . The samples were subjected to tension until the maximum force was reached, and the test was stopped. Then, a part of the sample was cut out from the area near to the notch. Finally, the material porosity defined by  $f_c$  parameter was assessed using the quantitative image analysis of the steel structure. The mean value of critical void volume fraction was determined as  $f_c = 0.0634$ .

The values of mean strain for void nucleation  $\epsilon_N$  were assumed also basing on the data applied by other authors for metallic materials and the results of analysis performed by Kossakowski and Wciślik [8] for S235JR steel. In the first step the value  $\epsilon_N = 0.3$  commonly found in the literature was used. Secondly the values  $\epsilon_N = 0.206$  was applied, basing on the experimental-numerical investigations of nucleation strain determined in analysis of distribution of stresses and strains around the  $Fe_3C$  particle in S235JR steel subjected to multi-axial stress state in the case of stress triaxiality  $\sigma_m/\sigma_e = 0.739$  [8].

The other parameters of the GTN model were assumed basing on the values determined by Kossakowski [5-7]. Finally, two sets of GTN parameters for S235JR steel were considered: set No. 'GTN 1' based on the literature data and set No. 'GTN 2' taking into consideration values of  $f_c = 0.0634$  and  $\epsilon_N = 0.206$  determined exactly for tested material. All GTN parameter considered in the analysis are summarized in **Table 1**.

**Table 1** Parameters of the GTN material models considered for S235JR steel [5-7]

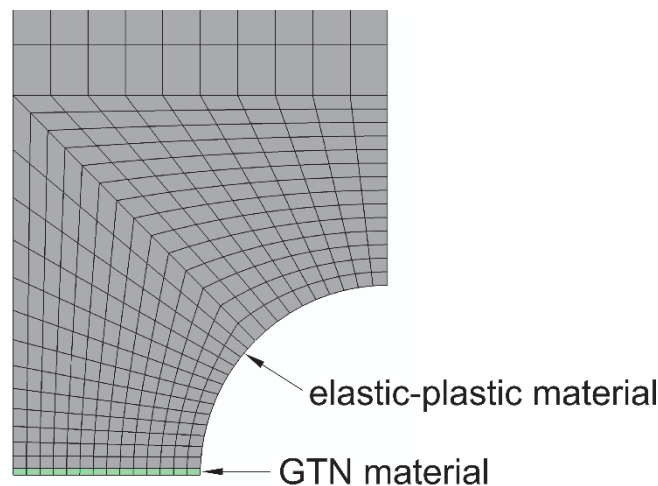
Set No.	$f_0$	$f_c$	$f_F$	$q_1$	$q_2$	$q_3$	$\epsilon_N$	$f_N$	$s_N$
GTN 1	0.0017	0.06	0.667	1.90	0.81	3.61	0.3	0.04	0.05
GTN 2	0.0017	0.0634	0.667	1.90	0.81	3.61	0.206	0.04	0.05

## 5. LOCAL GURSON-BASED MODELLING OF FAILURE IN S235JR STEEL

The method of local modeling of failure described in the introduction section was directly implemented during performed analysis. The scope of investigations covered numerical simulations of experimental tests of notched samples (see **Fig. 2**) subjected to tension in spatial stress state, described in detail in section 3.3.

Numerical calculations were performed using Abaqus v. 6.10 finite element method-based code. Taking into consideration the element symmetry, the axial-symmetric model was used.

During the analysis the process zone in the area subjected to the failure was modeled. In this region the Gurson-based material model for porous media, i.e. the GTN material model was applied. It was assumed in the region directly adjacent to the crack plane, as can be seen in **Fig. 3**.



**Fig. 3** Fracture zone modeled locally in numerical model of analyzed specimen

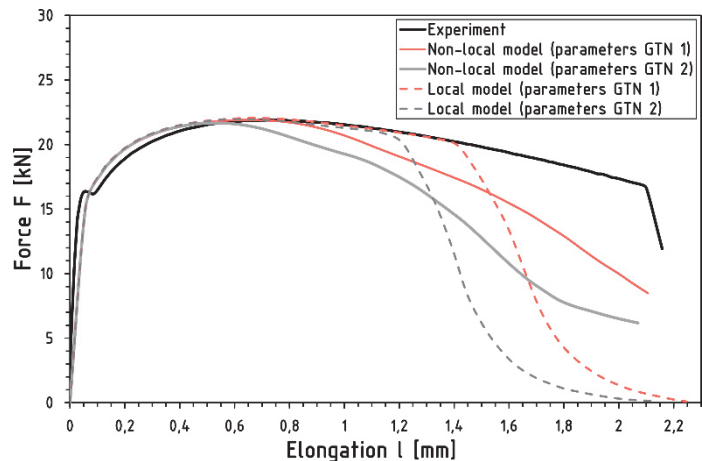
Thickness of the porous-media layer was established basing on the results obtained by Kossakowski [5], i.e. on the so called characteristic length  $l_c$ , defining the minimal dimensions of finite element mesh necessary to simulate failure of material. For S235JR steel  $l_c$  parameter was determined as  $l_c = 250 \mu\text{m}$  [5]. Taking into consideration the element symmetry, the thickness of the process zone was  $125 \mu\text{m}$ . For the rest of the numerical model elastic-plastic material was assumed.

Comparative analysis was also performed, assuming the GTN material for whole numerical model.

## 6. ANALYSIS OF THE $f_c$ AND $\epsilon_N$ PARAMETERS IN LOCAL GURSON-BASED MODELLING OF FAILURE OF S235JR STEEL AT HIGH STRESS TRIAXIALITY

Performed analysis was based on force-elongation  $F(l)$  curves determined numerically basing on local modeling of failure (local-models) for GTN parameters listed in **Table 1**. The results were compared to those obtained numerically for models for whom GTN material model was assumed for whole numerical models (non-local models). The experimental data were used as reference values (see **Fig. 4**).

**Fig. 4** Force-elongation curves determined experimentally and numerically



As can be seen from the results obtained, for all cases simulated numerically, the convergence in force-elongation curves is observed in relation to the curves determined experimentally from the beginning up to the about maximal force  $F$ . After that, for non-local failure modeling, smooth decrease in forces is noticed, up to the failure. For the numerical models where local failure modeling was used, it is noticed convergence in  $F(l)$  curve for subsequent range, after load-carrying capacity of elements is reached. The sudden drop in  $F(l)$  curves is observed in further stage of deformation, unlike as for non-local models.

Another phenomenon is visible effect of critical void volume fraction  $f_c$  and mean strain for void nucleation  $\varepsilon_N$  assumed during failure simulation of S235JR steel at high stress triaxiality. When applied values of  $f_c = 0.0634$  and  $\varepsilon_N = 0.206$  determined for S235JR steel, earlier initiation of failure process, manifested by earlier decrease of force-elongation  $F(l)$  curves, was observed in comparison to the values  $f_c = 0.06$  and  $\varepsilon_N = 0.3$  based on the literature data. For  $f_c = 0.0634$  and  $\varepsilon_N = 0.206$  much conservative results of failure simulations are observed in comparison to the typical values based on the literature data.

## CONCLUSIONS

During the S235JR steel failure modeling in high stress triaxiality, following phenomena were observed:

- when micro-damages were modeled in the material structure by means of the process zone based on GTN material model for porous media, the better convergence of strength curves determined numerically and experimentally is noticed than for models when porous material is assumed globally for whole element,
- significant effect of critical void volume fraction  $f_c$  and mean strain for void nucleation  $\varepsilon_N$  on strength of the material was observed; for values  $f_c = 0.0634$  and  $\varepsilon_N = 0.206$  determined for S235JR steel, the initiation of material failure was noticed visible earlier than for values  $f_c = 0.06$  and  $\varepsilon_N = 0.3$  based on literature data.

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