

## NUMERICAL DETERMINATION OF CRITICAL VOID NUCLEATION STRAIN IN THE GURSON-TVERGAARD-NEEDLEMAN POROUS MATERIAL MODEL FOR LOW STRESS STATE TRIAXIALITY RATIO

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### Abstract

The article describes an example of the numerical determination of the mean void nucleation strain which is the basic parameter of the Gurson-Tvergaard-Needleman (GTN) porous material model. The investigation was conducted for S355 steel, commonly used in civil engineering. Using criterion for the matrix-inclusion interface decohesion, developed by Argon et al, a numerical simulation of the void nucleation process was conducted, paying special attention to determine the values of stress and strain accompanying the formation of a void in a low level of stress state triaxiality ratio equal to 0.516. Axisymmetric model of rigid particle in the elastic-plastic matrix was used. The geometric parameters of the particles were adopted on the basis of microstructural analysis of the material. It was assumed that the critical radial stress at the interface between the particle and the matrix defines the moment of the void formation. The approach is different from that used by many authors, who set the GTN model parameters by adjusting the results of numerical simulation to experimental data. Critical strain equal to 0.2867 was obtained during the simulation. The parameter was used for the numerical simulation of tensile test of specimens with a ring notch. The results were compared to the force-elongation curves obtained experimentally for the specimens with the notch. The results of simulation using the determined value of nucleation strain were in good agreement with the experimental data.

**Keywords:** Ductile fracture, Gurson-Tvergaard-Needleman (GTN) model, numerical simulation, microvoid

### 1. INTRODUCTION

Simulation of inelastic behavior of a structural element, operating under pre-failure conditions is still relatively complicated engineering issue. This type of analysis is relatively simple in the case of uniaxial stress states. However effort estimation for a structural member working in a complex stress state (described by three principal stresses) is much more complicated. In that case reduced stresses are calculated according to different hypotheses. The stresses are then compared to the material characteristics. Different types of materials require the use of different hypotheses. For steel structures the Huber-Mises-Hencky (HMH) criterion is usually used. It has been demonstrated that the scope of the classical HMH solution use is restricted to the elastic range. It should be also noted that the most of the plasticity models does not allow the description of the plastic behavior in the range between the maximum load (material strength) and failure. This range is characterized by a gradual decrease in the load (the so called softening).

The main drawback of many plasticity models is the lack of a parameter responsible for the description of material softening after reaching its strength. This is due to the assumption of material continuum that does not allow for modeling the microdamage development, which is responsible for the material softening. The Gurson-Tvergaard-Needleman (GTN) porous material model introduces a softening parameter in the form of volume fraction of voids and second phase particles. The description of voids growth allows the simulation of material softening in the final phase of the loading process.

In the present article an attempt is made to determine numerically the value of the mean void nucleation strain, which is the basic parameter of the GTN model.

## 2. GURSON-TVERGAARD-NEEDLEMAN (GTN) MATERIAL MODEL

Among the porous material models the most commonly cited in the literature is the Gurson model [1], developed by Tvergaard [2] and Tvergaard and Needleman [3], widely known as the Gurson-Tvergaard-Needleman (GTN) model. It combines the macroscopic flow and the description of increase in the volume fraction of voids, which is a parameter of the microstructure of the material. The void volume fraction is defined as the ratio of the current microvoids volume to the volume of the sample, according to the formula:

$$f = \frac{V_V}{V} \quad (1)$$

where:  $V_V$  - current void volume fraction,  $V$  - the sample volume

The GTN constitutive model is described by the following relationship:

$$\Phi = \left( \frac{\sigma_e}{R_e} \right)^2 + 2q_1 f^* \cosh \left( \frac{3q_2 \sigma_m}{2R_e} \right) - 1 - q_3 f^{*2} = 0 \quad (2)$$

where:  $\sigma_e$  - reduced stress according to the HMH criterion,  $R_e$  - yield stress,  $q_i$  - Tvergaard coefficients,  $\sigma_m$  - hydrostatic stress,  $f^*$  - actual void volume fraction, defined as follows:

$$f^* = \begin{cases} f & \text{for } f \leq f_c \\ f_c + \frac{\bar{f}_F - f_c}{f_F - f_c} (f - f_c) & \text{for } f_c < f < f_F \\ \bar{f}_F & \text{for } f \geq f_F \end{cases} \quad (3)$$

where:  $f_c$  - critical void volume fraction corresponding to the onset of microvoids coalescence,  $f_F$  - critical void volume fraction at the moment of failure,  $\bar{f}_F = \frac{q_1 + \sqrt{q_1^2 - q_3}}{q_3}$

Current void volume fraction is described by the relation:

$$\dot{f} = \dot{f}_{gr} + \dot{f}_{nucl} \quad (4)$$

where:  $\dot{f}_{gr}$  - increase in porosity due to growth of voids existing in the material,  $\dot{f}_{nucl}$  - increase in porosity due to nucleation of new voids

Voids growth is defined as follows:

$$\dot{f}_{gr} = (1-f) \dot{\epsilon}_p : I \quad (5)$$

where:  $\dot{\epsilon}_p$  - plastic strain rate tensor,  $I$  - second order unit tensor

Porosity growth corresponding to the nucleation of new voids is expressed by the relationship:

$$\dot{f}_{nucl} = \frac{f_N}{s_N \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\epsilon_{em}^{pl} - \epsilon_N}{s_N} \right)^2 \right] \dot{\epsilon}_{em}^{pl} \quad (6)$$

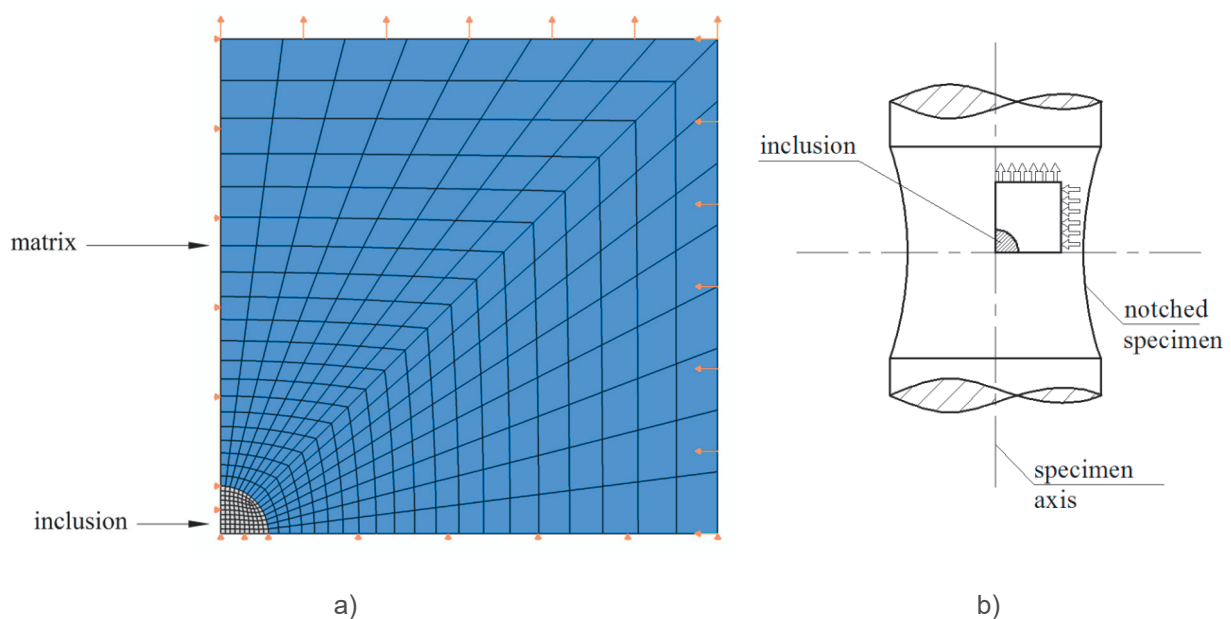
where:  $f_N$  - volume fraction of nucleating voids,  $\epsilon_N$  - mean nucleation strain,  $s_N$  - standard deviation of the nucleation strain,  $\epsilon_{em}^{pl}$  - equivalent plastic strain of the matrix,  $\dot{\epsilon}_{em}^{pl}$  - rate of the equivalent plastic strain of the matrix

As has been shown in several studies, the mean value of the nucleation strain strongly influences the simulation results using the GTN model. The widely accepted value of  $\epsilon_N$  is 0.3. Adoption of such a value is not, however, justified by the results of microstructural studies or simulations of void nucleation phenomena. The goal of the present article is to assess the value of  $\epsilon_N$  for low stress triaxiality ratio, using numerical simulation of void nucleation process.

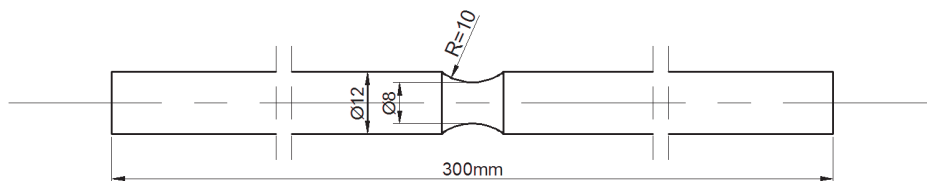
### 3. METHODOLOGY FOR THE $\epsilon_N$ PARAMETER DETERMINATION

A numerical simulation of the void initiation process was performed. The study aimed to determine the moment of voids initiation at inclusions in the investigated S355J2G3 steel under complex stress state, taking into account the calculation of average strain  $\epsilon_N$  necessary to initiate the void.

Axisymmetric model of a rigid inclusion in the elastic-plastic matrix was built (Fig. 1a). The inclusion was assumed to be separated from the specimen with a ring notch (Fig. 1b and Fig. 2).



**Fig. 1** a) numerical model of a rigid inclusion in elastic-plastic matrix used for the void initiation simulation [4],  
b) the position of the analyzed inclusion in the specimen with a ring notch



**Fig. 2** Specimen with a ring notch modeled in the study

Abaqus 6.10 software was used. The geometric parameters of the particles were adopted on the basis of microstructural examinations of the investigated S355J2G3 steel [5]. A spherical shape of inclusion and its

diameter equal to 2  $\mu\text{m}$  were adopted. Volume fraction of the inclusion was equal to 0.0009, i.e. the maximum value of the volume fraction of voids and the second phase particles, as determined during the microscopic examinations [5]. As the microstructural investigations revealed the presence of  $\text{Fe}_3\text{C}$  particles in S355J2G3 steel, this type of inclusion was considered in the model. The high stiffness of the inclusion material was achieved by setting a high value of Young's modulus (300 GPa). Furthermore it was assumed that the elastic-plastic properties of the matrix are characterized by the strength parameters of S355J2G3 steel.

It was assumed that the both phases are connected only on the upper half of the surface of their contact.

The boundary conditions were adopted in order to prevent vertical displacement of the edge perpendicular to the loading direction (horizontal lower edge in **Fig. 1a**) and the horizontal displacement of the edge lying on the longitudinal symmetry axis (the left vertical edge in **Fig. 1a**). Due to the symmetry, only one quarter of the inclusion and the adjacent matrix was modeled. Another issue is the adoption of an appropriate criterion for the void formation. It was assumed that the primary mechanism causing the cavity initiation was decohesion of the inclusion-matrix connection. According to the results described by Argon et al [6], the formation of voids takes place when the radial stress at the inclusion-matrix interface reaches the critical value. The radial interfacial stress was calculated according to the formula proposed by Argon et al [6]:

$$\sigma_{\text{rr}} = \sigma_{\text{m}} + \sigma_{\text{e}} \quad (7)$$

where:  $\sigma_{\text{rr}}$  - radial stress on the inclusion-matrix border,  $\sigma_{\text{m}}$  - hydrostatic stress,  $\sigma_{\text{e}}$  - equivalent stress according to the Huber-Mises-Hencky criterion

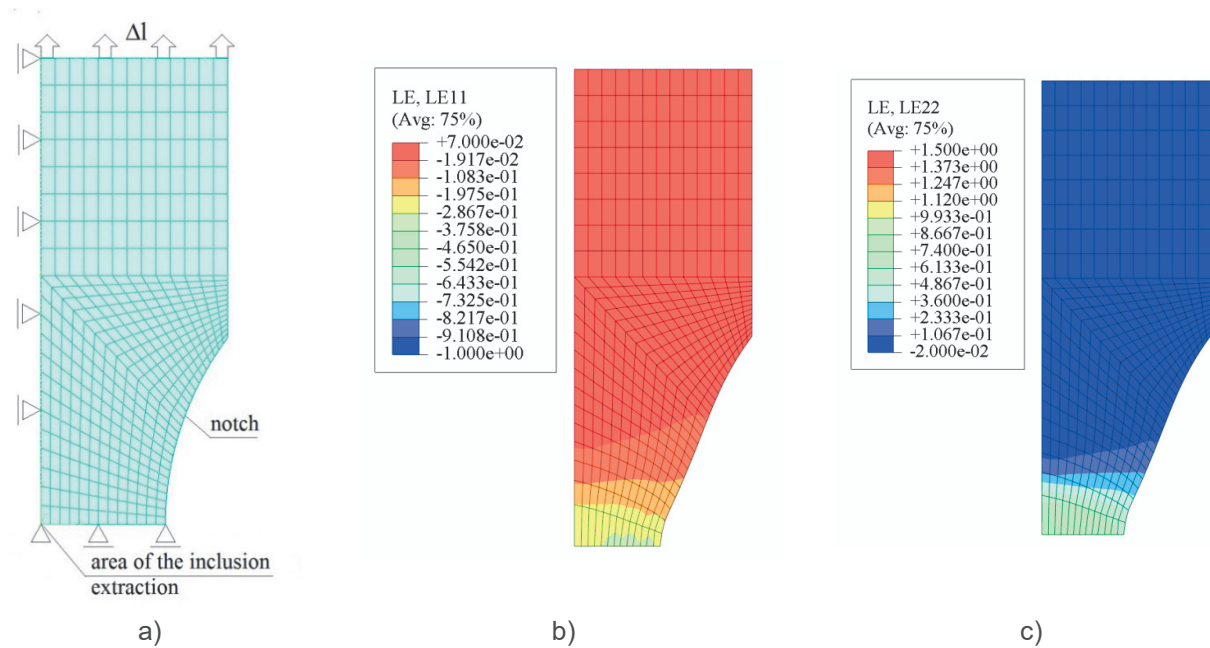
The adopted criterion for the void formation on  $\text{Fe}_3\text{C}$  particle can be expressed as follows [6]:

$$\sigma_{\text{rr}} = \sigma_{\text{rr}}^{\text{crit}} = 1670 \text{ MPa} \quad (8)$$

where:  $\sigma_{\text{rr}}^{\text{crit}}$  - critical radial stress on the inclusion-matrix border

Load was applied by constant, controlled displacement growth. The values of displacement were calculated in order to provide the global strain of the model (defined as an extension/shortening of the model in a given direction) equal to the corresponding strain of notched specimen in its axis (**Fig. 1b**). Thus, it was assumed that the analyzed inclusion was extracted from the center of the notched specimen (**Fig. 1b**). Due to the lack of experimental data, the strains in the notched specimen center were determined by the numerical simulation of the specimen in tension. Abaqus 6.7 software was used for simulation. The specimen was modeled as an axisymmetric element. Due to the symmetry, only one quarter of the specimen was modeled. The model height (30 mm) was equal to the half of the base length of the extensometer that was used during the experimental investigations. Other dimensions were adopted according to the geometry of the specimen - the minimal diameter was equal to 8 mm, the maximal diameter beyond the notch was 12 mm (**Fig. 2**). In the notch area the mesh size equal to 0.3 mm was used. FE mesh size was determined on the basis of microstructural analysis of the material as equal to the average distance between large inclusions. The notched specimen numerical model is shown in **Fig. 3a**.

The loading process was simulated by controlled growth of displacement which was applied to the upper edge of the model (identified with the place of extensometer mounting during experimental test). Because only one quarter of the specimen was modeled, the displacement value was equal to the half of the displacement recorded by the extensometer during experimental tensile testing. Elastic-plastic material model of S355J2G3 steel was used [5]. Maps of plastic strain in the notched specimen before failure are shown in **Fig. 3b** (strain in the direction perpendicular to the specimen axis) and **Fig. 3c** (strain parallel to the specimen axis). As stated previously, the inclusion was assumed to be extracted from the specimen center. Thus, strains calculated in that point were used for void initiation analysis: 0.825 for the deformation along the specimen axis and -0.261 for the radial deformation (perpendicular to the specimen axis).

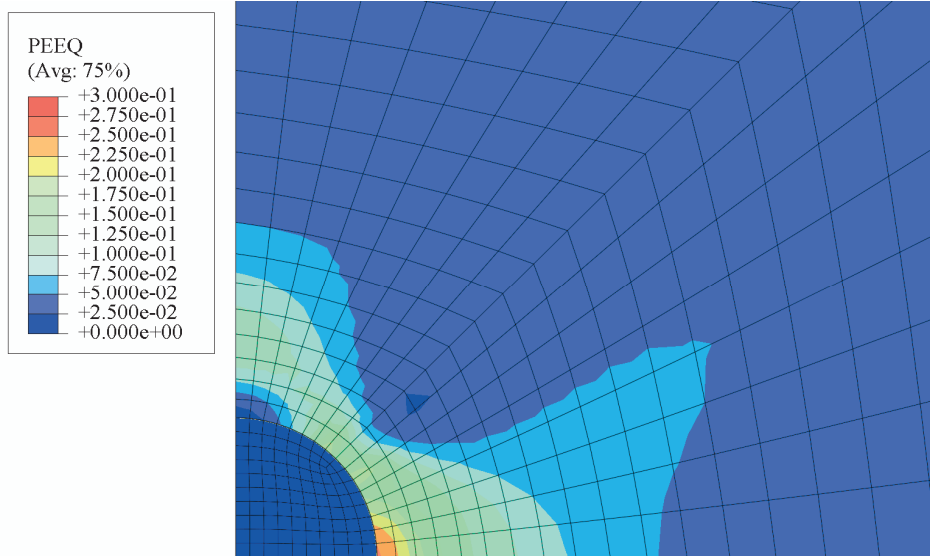


**Fig. 3** a) FEM model of the specimen with a notch, b) map of radial strain (perpendicular to the specimen axis) before the failure, c) map of longitudinal strain (parallel to the specimen axis) before the failure

#### 4. RESULTS OF THE VOID INITIATION SIMULATION

Using the inclusion numerical model, described in the previous section, and strains obtained during the simulation of the tensile notched specimen, a simulation of a void initiation process was performed. Interfacial decohesion was adopted as a mechanism for  $Fe_3C$  inclusion and matrix separation. A stress criterion for the inclusion separation, established by Argon et al [6], was used in simulation. The critical radial interfacial stress was 1670 MPa. It was assumed that the maximal plastic strain in the model at the moment of decohesion criterion fulfillment is equal to critical strain for void initiation  $\epsilon_N$ , which is the basic parameter of the GTN model.

**Fig. 4** illustrates the map of equivalent plastic strain around the inclusion at the moment of void initiation, in accordance with the adopted criterion. A characteristic feature is the presence of maximum plastic deformation in the area adjacent to the inclusion symmetry axis perpendicular to the loading direction (the lower horizontal edge in **Fig. 4**). The largest strain (identified with the void nucleation strain) was  $\epsilon_N=0.2867$ . Due to the high inclusion stiffness, the strains in its region were 0.

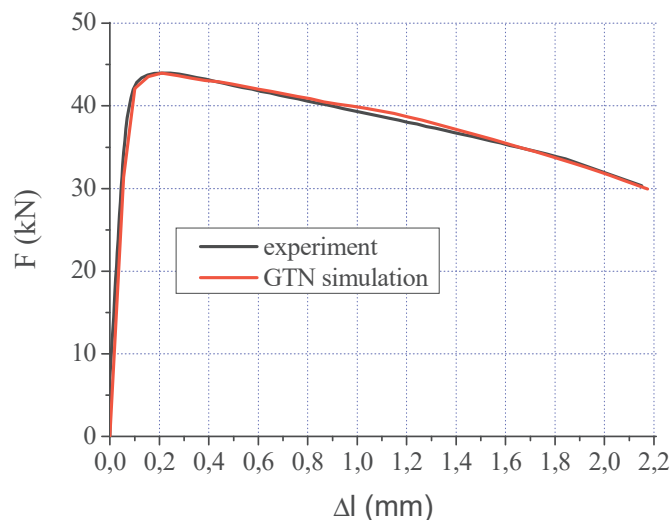


**Fig. 4** Map of equivalent plastic strain around the Fe<sub>3</sub>C inclusion at the moment of void initiation

## CONCLUSIONS

The obtained value of the average nucleation strain  $\epsilon_N=0.2867$  was in very good agreement with the value commonly used in the literature  $\epsilon_N=0.3$ .

The GTN model and the value of its parameter  $\epsilon_N=0.2867$ , obtained in this study, were used to simulate the tensile test of the notched specimen. The numerical model of the specimen, described previously, was used. The model was modified by using the GTN criterion and the obtained value of  $\epsilon_N=0.2867$  parameter. The other GTN parameters were adopted according to [5]. **Fig. 5** illustrates the simulated force-displacement curve compared with the curve obtained during experimental test. A good agreement of the results, as well as the possibility to simulate the material softening by the GTN model, were observed.



**Fig. 5** Force-elongation curves for the specimen with notch radius, obtained experimentally and numerically, using the GTN model and the void nucleation strain  $\epsilon_N=0.2867$  (according to the present study)

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