



## EXPERT SYSTEM BASED SUPPORT TO SANDWICH PALLET LOADING PROBLEM IN THE FMCG DISTRIBUTION COMPANY

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### Abstract

This paper presents a multi-pallet loading problem, assuming that products are heterogenous, and pallet loading units can be stacked creating sandwich (SNDW) loading units. The research task is to support the optimal composition of products on pallet units using the available space at the maximum level, and fulfilling the constraints, such as the maximum weight, height and stability of multi-pallet units. The aim of the research is to create the minimum number of SNDW pallets under the specific constraints of multi-pallets, products and a distribution process. The paper discusses the procedure for creating SNDW pallets with the application of simulation-based approach enriched with an original knowledge exploration procedure using Dominance-based Rough Sets Theory (DRST) and resulted rules generation. A simulation environment has been applied to verify the proposed approach, and its implementation has been carried out in a distribution company from the FMCG industry. The computational experiments have been carried out on a set of 700 picking orders. On that basis a set of decision rules has been created and used to evaluate new picking orders resulting in 83% effectiveness of their classification for potential improvement within multi-pallet loading process.

**Keywords:** Pallet Loading Problem (PLP), Mix Product Pallet (MIX), Sandwich Pallet (SNDW), Expert System, Dominance-based Rough Sets Theory (DRST)

### 1. INTRODUCTION

In the scientific literature, the problem addressed in this paper is called the Pallet Building Problem (PBP) or Pallet Loading Problem (PLP) [2, 4]. In solving this problem, the aim is to load a given set of products onto one or more pallets, satisfying general and specific constraints to minimise the number of pallets used. In practice, the authors of this paper face a family of the above issues, with the two most important cases being the loading of homogeneous and mixed pallets. A homogeneous pallet contains only the same SKUs or units with identical product attributes (e.g. batch number or expiration date), while a mixed pallet contains different products. In practice, the problem of stacking pallet load units (Stacked Pallet Loading Problem – SPLP) while meeting customer constraints and requirements has also emerged [1, 8]. The result is the building of load units called stacked pallets. These are also known as sandwich pallets and are a composition of a stack of pallets prepared for shipping or storage. This method is mainly used in the food/retail industry, where it is often a customer requirement to deliver in this way. Once such a pallet is received by the end customer, it can more easily separate the different pallets, and they can be used or stored immediately. Building load units in this way saves space in trucks and warehouses and thus contributes to a higher vehicle load factor and storage capacity. It also makes warehouse operations more efficient, as the forklift operator can carry two or even three pallets at a time. Efficiently stacking loaded pallets is crucial for optimizing storage space, reducing transportation costs, and ensuring the safe handling of goods.



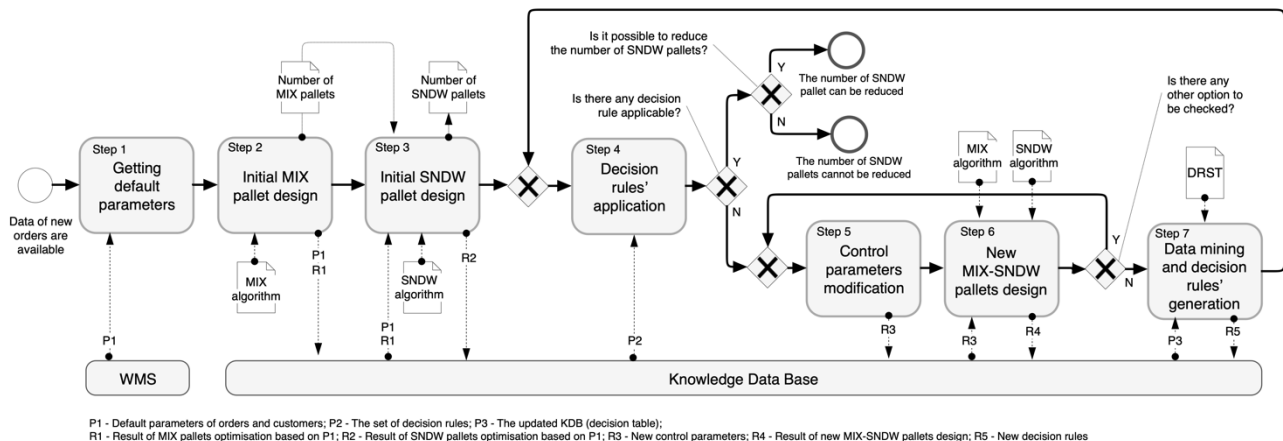
Both the PLP and SPLP problems are generalisations of the Bin Packing Problem and belong to NP-hard combinatorial optimisation problems [2, 4, 6, 9]. There are several papers in the literature that present different methods to solve these problems (exact, heuristics, meta-heuristics, simulation-based, AI-based), but the vast majority of them deal with different variants of the PLP problem. Relatively few works address practical aspects that take into account the real-world constraints and requirements of building sandwich loading units.

The research findings presented in this paper attempt to fill this gap. A hybrid decision model has been developed in which an inference module using rough set theory [3, 7] is an essential element. The result of this module is the extraction of relevant features that affect the minimisation of the number of SNDW loading units built.

## 2. THE PROPOSED METHODOLOGY

### 2.1 The proposed procedure

The proposed procedure is composed of 7 steps (Figure 1). It is assumed that all input data are available from Warehouse Management System (WMS). The orders to be completed on a daily basis are planned in advance, i.e., before the operation procedures on assembling products on pallets are started in the warehouse.



**Figure 1** The key steps of a proposed methodology

The default parameters (P1) of each order and information on customers are downloaded from WMS system at the beginning of the procedure (Step 1). Next, using optimisation algorithm of MIX pallets design (Step 2), the minimum number of pallet load units is defined (R1). This result together with P1 are saved in the Knowledge Data Base - KDB as a single record. The initial MIX pallets design is then analysed with respect to minimum number of SNDW pallets to be defined in Step 3. It is supported with an optimisation algorithm of SNDW pallets design, and the minimum number of these pallets (R2) is added to the record of MIX pallets in the KDB. It is a reference record with the optimal number of MIX pallets and SNDW pallets based on P1. The result of Step 3 is next evaluated (Step 4) using the set of decision rules (P2) generated upon data collected in KDB. This assessment can lead to two different situations. The first one is when the existing decision rules can be applied in the considered case. Then, there are two possible results of evaluation, i.e., the number of SNDW pallets can be reduced by changing selected P1, or the number of SNDW pallets cannot be reduced, i.e., changing selected P1 brings no improvement. The second situation is when it is not possible to apply the current set of decision rules in the considered case. Therefore, the selected control parameters, e.g., maximum acceptable weight/height of MIX and SNDW pallets, can be modified to some extent (Step 5). These new control parameters are added to KDB as a new record (R3). The customer's order with the modified control parameters is next optimised using MIX and SNDW pallets algorithms (Step 6). A resulted number of SNDW pallets is compared with the number of SNDW pallets from the reference record (R2). The sequence of Steps



5 and 6 is iteratively repeated until all possible changes of control parameters are verified. The new result of MIX pallets and SNDW pallets design is recorded in KDB (R4). Finally, in Step 7, data mining process based on the decision table in KDB is carried out (P3). This step is supported with a Dominance-based Rough Sets Theory – DRST. Based on its output, the set of decision rules is generated and its update is recorded in KDB (R5). The procedure of decision rules' application is repeated when a new case is analysed (Step 4).

## 2.2 Mathematical formulation

For each order, the number of SNDW pallets  $L_k$  resulted from each  $k$ -iteration (where  $k = 1, \dots, K$ ) of Steps 5 and 6 (Figure 1) is compared with the reference solution  $L_0$  being an output of Step 3. The maximum difference between values  $L_0$  and  $L_k$  (for  $k = 1, \dots, K$ ) is identified and based on that the most probable state of SNDW pallets potential reduction is defined (see the set of states  $\{I, N, R\}$ ) in equation (1).

$$PNC = \begin{cases} I & \text{if } \max_k(L_0 - L_k) < a, \\ N & \text{if } a \geq \max_k(L_0 - L_k) < b, \\ R & \text{if } \max_k(L_0 - L_k) \geq b, \end{cases} \quad (1)$$

where:  $I$  – the number of SNDW pallets can only be increased upon parameter's modification;  $N$  – the probability of the number of SNDW pallets reduction is very low;  $R$  – it is probable to reduce the number of SNDW pallets upon parameter's modification;  $L_0$  – the number of SNDW pallets in the reference record;  $L_k$  – the number of SNDW pallets in the  $k$ -iteration,  $k = 1, \dots, K$ ;  $a$  – the threshold value below which the number of SNDW pallets resulted from the optimisation based on modified parameters will increase;  $b$  – the threshold value for and above which the number of SNDW pallets resulted from the optimisation based on modified parameters will be reduced.

## 2.3. Rough Sets Theory

The key part of the methodology proposed in chapter 2.1 is based on the concept of Dominance-based Rough Sets Theory - DRST **Chyba! Nenalezen zdroj odkazů., Chyba! Nenalezen zdroj odkazů..** Formaly, it uses a 4-tuple information table  $S = \langle U, Q, V, f \rangle$ , where  $U$  is a finite set of *objects*, i.e., rows of the table,  $Q = \{q_1, \dots, q_m\}$  is a finite set of *characteristics*, i.e., columns of the table;  $V$  is the domain of characteristics  $q$ , expressed as  $V = \bigcup_{q \in Q} V_q$ ; and  $f$  is the information function assigned to each pair: object  $x$  – characteristic  $q$ , such that  $f : U \times Q \rightarrow V$  and  $f(x, q) \in V_q, \forall q \in Q, x \in U$ . Among the characteristics from  $S$  a conditional part  $C$  and a decision part  $D$  can be distinguished, i.e.,  $C \cup D = Q$ . Since the conditional part is composed of criteria  $C^>$  and attributes  $C^=$ , i.e., characteristics with preference ordered and non-ordered domains, respectively, see  $C^> \cup C^= = C$  and  $C^> \cap C^= = \emptyset$ , the information table is formally a decision table  $S = \langle U, C^> \cup C^= \cup D, V, f \rangle$ .

For two objects  $x, y \in U$ , where  $x$  represents reference objects and  $y$  compared objects, one can specify a binary relation  $R_p$  defined on characteristics  $P \subseteq C$ , resulted from the dominance  $D_q$  and indiscernible  $I_q$  functions, such as **Chyba! Nenalezen zdroj odkazů.:**

$$yR_px, \forall x, y \in U \text{ if } \begin{cases} yD_qx & \forall q \in P^> : P^> = P \cap C^> \\ yI_qx & \forall q \in P^= : P^= = P \cap C^= \end{cases} \quad (2)$$

Considering any object  $x \in U$  and relation  $R_p$  one can define a set  $R_p^+(x)$  of objects dominating  $x$ , i.e.,  $R_p^+(x) = \{y \in U : yR_px\}$ . Similarly,  $R_p^-(x)$  constitutes a set of objects dominated by  $x$ , i.e.,  $R_p^-(x) = \{y \in U : xR_py\}$ . All decision characteristics  $D$  constitute the partition of  $U$  into  $t$ -ordered categories  $\mathcal{Cl} = \{Cl_t, t \in T\}, T = \{1, \dots, n\}$ . In this sense objects assigned to class  $Cl_r$  are preferred against objects in category  $Cl_s$  if  $r > s, \forall r, s \in T$ . If an object belongs to class  $s$  according to  $D$  and its conditional part  $C$  outperforms conditional part of any other object assigned to class  $r$ , it generates a certain inconsistency.



Taking into account  $P \subseteq C$  and  $t \in T$  object  $x \in U$  belongs without any doubt to the upward union of classes  $Cl_t^{\geq}$  if  $R_p^+(x) \subseteq Cl_t^{\geq}$ , and a set of all such objects constitutes its lower approximation  $\underline{P}(Cl_t^{\geq})$ . On a contrary, the set of objects that probably belongs to  $Cl_t^{\geq}$  constitutes the upper approximation  $\overline{P}(Cl_t^{\geq})$ ; the comparison of upper and lower approximations results in the boundary region -  $Bn_p$ . More information on the DRST and a detailed description of decision rule induction is presented in **Chyba! Nenalezen zdroj odkazů.** and **Chyba! Nenalezen zdroj odkazů.**, respectively.

### 3. THE EXPERIMENTAL APPLICATION OF THE METHODOLOGY

#### 3.1. General assumptions and input data for the experimental testing

The experimental test of the procedure is two-fold. First, the procedure is tested on the data exploration and rule generation as a basis for initial knowledge to be ready for a decision support of new orders upcoming. Second, the decision-making effectiveness is tested, i.e., the level of new order support using the available knowledge.

The test of the procedure is performed on the data set extracted from the Warehouse Management System (WMS) of the FMCG distribution company. This data set is randomly selected, and its components are related to each *Order\_ID* of 729 picking orders applicable to construct MIX and SNDW pallets. The orders are composed of information of products to be completed, such as their type, number, and parameters, like weight and height; and default control parameters applied to the algorithms of MIX and SNDW pallets composition. The set of orders is divided into two subsets *A* and *B*. The subset *A* composed of 90% of orders, i.e., 665 picking orders, is selected for data exploration and rules generation. The subset *B* with the rest of picking orders, i.e., 64 (10%), is selected for testing the effectiveness of the decision support using the decision rules.

#### 3.2. Data exploration and rules generation

To perform the test on the data exploration and rules generation the sequence of steps is simulated. In Step 1 (Figure 1) the data subset *A* of 665 picking orders is extracted from the WMS. In Steps 2 and 3, for each *Order\_ID* from *A* and with their default control parameters P1 the sequence of MIX and SNDW optimisation algorithms are applied. An exemplary record 1 out of 665 resulted from Steps 1-3 is presented in Figure 2.

Example_1: 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 4, <b>1450, 900</b> , 0, 0, 0, 0, 0, 0, <b>10</b> , 0, 0, <b>0.7</b> , <b>1, 1800, 750</b> , 4, 0, d, 7845719
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**Figure 2** An example of one record from subset *A* extracted from KDB after Steps 1-3

Each record is identified by the *Order\_ID* (the last value in Figure 2). The record is composed of 41 conditional characteristics and P1 are bolded, i.e.: *Max\_MIX\_height* (1450 mm); *Max\_MIX\_weight* (900 kg); tolerance of the difference between product height on each layer of the MIX pallet (10 mm); the minimum degree (0.7=70%) and the maximum degree (1=100%) of filling the last layer with products of the MIX pallet, i.e., on this MIX pallet can be placed another one, *Max\_SNDW\_height* (1800 mm), *Max\_SNDW\_weight* (750 kg). The decision part of the data set is identified, i.e. the last 4 characteristics in the record are as follows: a number of the SNDW pallets *N\_SNDW* (4 items), the result of  $L_0 - L_k$  (0, it is a reference record), identifier of the type of control parameters (d for default values), and the *Order\_ID* (7845719). Step 4 is applied when the decision rules are generated; it is skipped during the test on the data exploration and rules generation. In Step 5, for each *Order\_ID* the set of P1 is modified by a generation of new values as deviations from default settings. During the test, the modification of P1 has been limited to 2 items, i.e., *Max\_MIX\_height* and *Max\_MIX\_weight*. All combinations of the values have been analysed. The ranges of the modified parameters have been limited to  $\langle -0.3; 0.1 \rangle$  of *Max\_MIX\_height* and  $\langle -0.3; 0.1 \rangle$  of *Max\_MIX\_weight* with a step of variation 0.05. As a result, for each *Order\_ID* 81 iterative results have been generated (Step 6), i.e., 59,049 simulations have been run; and for each *Order\_ID* the decision characteristic  $D = PNC$  has been identified according to equation (1), see  $PNC = \{I, N, R\}$ . In Step 7, all records



from the subset  $A$  are explored to identify upper and lower approximations of the upward and downward union of classes (Table 1). The overall quality of approximation is 0.881, which is the acceptable level, and it also means that approximately 12% of records are ambiguous. With reference to Table 1 it can be seen that the union of classes with the lowest quality of approximation is the upward one being at least  $R$ , i.e.  $Cl_R^{\geq}$ :  $PNC \geq R$ .

**Table 1** Upper and lower approximation of union of classes  $I, N, R$

Item	Union of classes	Quality of approximation, (%)	Cardinality, (item)
1	At most $I$ , i.e. $PNC \leq I$	0.764	165
2	At least $N$ , i.e. $PNC \geq N$	0.911	499
3	At most $N$ , i.e. $PNC \leq N$	0.935	654
4	At least $R$ , i.e. $PNC \geq R$	0.140	10

Within Step 7 it is also possible to generate 336 reducts, and the *core* as a common part of them contains 7 out of 41 conditional characteristics. Finally, in Step 7, with support of VC-DomLEM algorithm **Chyba! Nenalezen zdroj odkazů.**, the set of decision rules is generated. It is based on the lower approximations of the union of classes. 129 decision rules are generated, and their main characteristics including examples are presented in Table 2.

**Table 2** The structure of generated decision rules

Item	Decision part	Conditional part (no. of cncls)	No. of rules	Exemplary rules
1	$PNC \geq R$	1-4	7	#2: if $C_9 \geq 1$ & $N\_SNDW \leq 0$ & $C_{25} \leq 0$ & $Max\_MIX\_weight \geq 1000$ then $PNC \geq R$ ;
2	$PNC \geq N$	1-3	50	#21: if $C_9 \geq 0.06$ & $C_{13} \geq 0.81$ then $PNC \geq N$ ; #58: if $C_6 \geq 0.19$ & $C_8 \geq 0.73$ then $PNC \geq N$ ;
3	$PNC \leq I$	1-7	46	#102: if $C_6 \leq 0$ & $C_7 \leq 0$ & $C_8 \leq 0$ & $N\_MIX \geq 1$ & $C_{25} \geq 2$ then $PNC \leq I$ ;
4	$PNC \leq N$	1-4	26	#127: if $C_6 \leq 0$ & $C_9 \leq 0$ & $Max\_MIX\_weight \leq 850$ then $PNC \leq N$

### 3.3. The application of decision rules to the decision making

To perform the test on decision rules application to the decision making different steps are simulated. In Step 1, from the WMS the subset  $B$  of 64 out of 729 *Order\_ID* is extracted. In Steps 2 and 3, for each *Order\_ID* the sequence of MIX and SNDW optimisation algorithms with their default control parameters P1 are applied. An exemplary record resulted from Steps 1-3 is presented in Figure 3; default control parameters are bolded.

Example_12070: 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 4, <b>1600, 750</b> , 0, 0, 0, 0, 0, 0, <b>10</b> , 0, 0, <b>0.7, 1, 1800, 750</b> , 1, 0, d, 7850102
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**Figure 3** An example of one record from subset  $B$  extracted from KDB after Steps 1-3

**Table 3** An overview of experimental test of decision rules application to classify 64 picking orders

Item	Result of classification	No. of records (orders)	Result, in (%)
1	Correct	54	83.1
2	Wrong	10	16.9
3	Unclassified	0	0



During Step 4 each *Order\_ID* is analysed with the application of decision rules from KDB (see P2 in Figure 1). For the exemplary record (Figure 3) a group of 6 decision rules extracted from KDB is specified and its conditional part is consistent with the conditional part of the analysed record, i.e. 1) **if**  $C_1 \leq 1$  **then**  $PNC \geq N$ ; 2) **if**  $C_{15} \geq 0.99$  &  $N\_FULL \leq 1$  **then**  $PNC \geq N$ ; 3) **if**  $C_1 \leq 2$  &  $C_9 \geq 0.3$  **then**  $PNC \geq N$ ; 4) **if**  $C_1 \leq 3$  &  $C_9 \geq 0.57$  **then**  $PNC \geq N$ ; 5) **if**  $Max\_MIX\_height \leq 800$  **then**  $PNC \geq N$ . Step 4 is repeated for all 65 records from subset *B*, and the overview of its result is presented in Table 3.

### 3.4. Discussion of the results

Based on the test results (Table 3) it can be concluded that more than 83% of analysed picking orders are classified correctly. It means that the proposed procedure guarantees the answer to the question whether the number of SNDW pallets can be reduced or not. For 17% of orders the wrong answer is generated, which means that it is not possible to classify them as for potential reduction and default parameters P1 should be applied.

## 4. CONCLUSION

In this paper the methodology of a decision support procedure on MIX and SNDW pallet design with the application of the simulation and DRST has been presented. Based on it, it is possible to create decision rules and classify new picking orders for the potential reduction of SNDW pallets. The proposed procedure has been applied on the data set from the FMCG company. The analysed case shows that more than 80% of new orders have been classified correctly, which means that there is a potential of SNDW pallets reduction. After successful implementation of the methodology, the future directions of this research are as follows: (1) the KDB should be extended with the decision rules generated on upcoming orders, resulting in a higher number of correct classifications, (2) further tests are necessary to be carried out, (3) the set of control parameters to be tested and modified should be extended, (4) the proposed decision support procedure should be verified on more case studies from different branches.

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